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B014

CSE-B

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Tutorial 1

$$Q1) \frac{(3+i)(1-i2)}{1+i}$$

$$\text{Let } z = \frac{3(1-i2) + i(1-i2)}{1+i}$$

$$= \frac{3 - i6 + i - i^2 2}{1+i}$$

$$= \frac{3 - i5 - (-)2}{1+i} = \frac{3 - i5 + 2}{1+i}$$

$$= \frac{(3-i) - 5-i}{1+i} \times \frac{1-i}{1-i}$$

$$= \frac{5(1-i)(1-i)}{1-i^2} = \frac{5(1-2i+i^2)}{1-(-1)}$$

$$= \frac{5(1-2i-1)}{2}$$

$$= \frac{5-10i-5}{2} = \frac{-10i}{2} = -5i$$

$$z = \boxed{-5i}$$

2)  $(1+i)^{-3}$

Let  $z = \frac{1}{(1+i)^3} = \frac{1}{(1^3 + 3(1)^2(i) + 3(1)(i)^2 + i^3)}$

$= \frac{1}{1+i^3-3-i}$

$= \frac{1}{-2+i2}$

$= \frac{1}{-2+i2} \times \frac{-2-i2}{-2-i2}$

$= \frac{-2-i2}{(-2)^2 - (i2)^2} = \frac{-2(1+i)}{4 - i^2 4} = \frac{-2(1+i)}{2 \cdot 4(1 - (-1))}$

$= \frac{-2(1+i)}{2(2)} = \frac{-1-i}{2}$

$z = \left[ \begin{matrix} -\frac{1}{2} & -\frac{i}{2} \\ \frac{1}{4} & \frac{i}{4} \end{matrix} \right]$

$$3) \frac{(1+i^3)^2}{(3-i)^3}$$

$$\text{let } z = \frac{(1+i^3)^2}{(3-i)^3} = \frac{(1-i)^2}{(3-i)^3} = \frac{1+i^2 - 2(1)(i)}{3^3 - i^3 - 3(3)(i)(3-i)}$$

$$= \frac{1-1-i2}{27+i-i^2 27-i^2 9}$$

$$= \frac{-i2}{27+i-i^2 27+i^2 9}$$

$$= \frac{-i2}{27-i26-9}$$

$$= \frac{-i2}{18-i26} = \frac{-i2}{18-i26} \times \frac{18+i26}{18+i26}$$

$$= \frac{-i2(18+i26)}{(18)^2 - i^2(26)^2}$$

$$= \frac{-i36 - i^2 52}{324 - (-1)676}$$

$$= \frac{-i36 - (-1)52}{324 + 676}$$

$$z = \frac{52 - i36}{1000} = \frac{52}{1000} - i \frac{36}{1000}$$

$$\therefore z = \frac{13}{250} - i \frac{9}{250}$$

$$4) \frac{1}{2-i3} + \frac{5-i}{6+i2}$$

$$\text{Let } z_1 = \frac{1}{2-i3} \times \frac{2+i3}{2+i3}$$

$$= \frac{2+i3}{4-i^2 9}$$

$$= \frac{2+i3}{4-(-1)9}$$

$$= \frac{2+i3}{13}$$

$$z_1 = \frac{2}{13} + i \frac{3}{13}$$

$$\text{Let } z_2 = \frac{5-i}{6+i2} \times \frac{6-i2}{6-i2}$$

$$= \frac{5(6-i2) - i(6-i2)}{36 - i^2 4}$$

$$= \frac{30 - i10 - i6 + i^2 2}{36 - (-1)4}$$

$$= \frac{30 - i16 + (-1)2}{36 + 4}$$

$$= \frac{30 - i16 - 2}{40}$$

$$= \frac{28 - i16}{40}$$

$$= \frac{28}{40} - i \frac{16}{40}$$

$$z_2 = \frac{7}{10} - i \frac{2}{5}$$

$$\therefore z_1 + z_2$$

$$\therefore z_1 + z_2$$

$$\begin{aligned} \therefore z_1 + z_2 &= \frac{2}{13} + i\frac{3}{13} + \frac{7}{10} - i\frac{2}{5} \\ &= \left(\frac{2}{13} + \frac{7}{10}\right) + i\left(\frac{3}{13} - \frac{2}{5}\right) \\ &= \left(\frac{2}{13} + \frac{7}{10}\right) + i\left(\frac{3}{13} - \frac{2}{5}\right) \end{aligned}$$

$$\therefore z_1 + z_2 = \boxed{\frac{111}{130} - i\frac{11}{65}}$$

$$5) \frac{4i^8 - 3i^9 + 3}{3i^{11} - 4i^{10} - 2}$$

$$\begin{aligned} \text{let } z &= \frac{4i^8 - 3i^9 + 3}{3i^{11} - 4i^{10} - 2} \\ &= \frac{4(i^2)^4 - 3(i)(i^2)^4 + 3}{3(i^2)^5(i) - 4(i^2)^5 - 2} \\ &= \frac{4(-1)^4 - 3(i)(-1)^4 + 3}{3(-1)^5(i) - 4(-1)^5 - 2} \\ &= \frac{4 - i3 + 3}{-i3 + 4 - 2} \\ &= \frac{7 - i3}{2 - i3} \\ &= \frac{7 - i3}{2 - i3} \times \frac{2 + i3}{2 + i3} \\ &= \frac{7(2 + i3) - i3(2 + i3)}{4 - i^2 9} \\ &= \frac{14 + i21 - i6 - i^2 9}{4 - (-1)9} \\ &= \frac{14 + i15 - (-1)9}{13} \\ &= \frac{14 + i15 + 9}{13} = \frac{23 + i15}{13} \end{aligned}$$

$$z = \frac{23}{13} + i \frac{15}{13}$$

$$6) \frac{2i^3 - 3i^2}{3i^2}$$

$$\text{let } z = \frac{2i^3 - 3i^2}{3i^2}$$

$$z =$$

$$6) \frac{2i^3 - 3i^7 + 4i^6 + 2}{3i^2 - 4i^5 + 4}$$

Let  $z = \frac{2i^3 - 3i^7 + 4i^6 + 2}{3i^2 - 4i^5 + 4}$

$$= \frac{2(i)(i^2) - 3(i)(i^2)^3 + 4(i^2)^3 + 2}{3(-i)^2 - 4(i)(i^2)^2 + 4}$$

$$= \frac{2(i)(-1) - 3(i)(-1)^3 + 4(-1)^3 + 2}{3(-1) - 4(i)(-1)^2 + 4}$$

$$= \frac{-2i + 3i - 4 + 2}{-3 - 4i + 4}$$

$$= \frac{i - 2}{-4i + 1} = \frac{-2 + i}{1 - 4i} \times \frac{1 + 4i}{1 + 4i}$$

$$= \frac{-2(1 + 4i) + i(1 + 4i)}{1 - i^2 16}$$

$$= \frac{-2 - i8 + i + i^2 4}{1 - (-1)16}$$

$$= \frac{-2 - i7 - 4}{17}$$

$$= \frac{-6 - i7}{17}$$

$$z = \frac{-6 - i7}{17}$$

Do as directed

7) If  $(x+iy)(2-i3) = 4-i$ ; find  $x$  &  $y$

$$\therefore (x+iy)(2-i3) = (4-i)$$

$$\therefore (x+iy) = \frac{(4-i)}{(2-i3)}$$

$$= \frac{(4-i) \times (2+i3)}{(2-i3) \times (2+i3)}$$

$$= \frac{4(2+i3) - i(2+i3)}{4 - i^2 9}$$

$$= \frac{8 + i12 - i2 - i^2 3}{4 - (-1)9}$$

$$= \frac{8 + i10 - (-1)3}{4 + 9}$$

$$= \frac{8 + i10 + 3}{13}$$

$$= \frac{11 + i10}{13}$$

$$x+iy = \frac{11}{13} + i \frac{10}{13}$$

$$\therefore \boxed{x = \frac{11}{13} ; y = \frac{10}{13}}$$

8) If  $(-)$

Let  $z =$

=

=

=

z

$\therefore x +$

$\therefore \boxed{x =$

8) If  $\left(\frac{1-2i}{3}\right)^3 = x+iy$ ; find  $x$  &  $y$

$$\text{Sol: } z = \left(\frac{1-2i}{3}\right)^3$$

$$= \frac{(1-2i)^3}{3^3}$$

$$= \frac{1^3 - (2i)^3 - 3(1)^2(2i) + 3(1)(2i)^2}{27}$$

$$= \frac{1 - 8(-i) - 6i + 12i^2}{27}$$

$$= \frac{1 + i8 - i6 - 12}{27}$$

$$= \frac{-11 + i2}{27}$$

$$z = \frac{-11 + i2}{27}$$

$$\therefore x+iy = \frac{-11}{27} + i\frac{2}{27}$$

$$\therefore \boxed{x = \frac{-11}{27} ; y = \frac{2}{27}}$$

9) Find  $x$  &  $y$  which satisfies the equation  $(3x-4y)^2 + i(x^2-y^2) = 5$

Sol<sup>n</sup>:-  $\therefore 3x-4y = 5$  — (1)  
 $x^2-y^2 = 0$   
 $(x+y)(x-y) = 0$  — (2)

If  $(x+y) = 0$ ,  
 Multiply eq by 4.  
 $\therefore 4x+4y = 0$  — (2)

Add (1) & (2),  
 $3x-4y = 5$   
 $(+)$   $4x+4y = 0$   
 $7x = 5$

$\therefore x = \frac{5}{7}$

put  $x = \frac{5}{7}$  in (1)

$3\left(\frac{5}{7}\right) - 4y = 5$

$\frac{15}{7} - 4y = 5$

$-4y = 5 - \frac{15}{7}$

$-4y = \frac{35-15}{7}$

$-4y = \frac{20}{7}$

$\therefore y = -\frac{5}{7}$

If  $(x-y)=0$  ,  
Multiply eq. by 4.

$$4x - 4y = 0 \quad \text{--- (3)}$$

Sub (3) From (1) ,

$$\begin{array}{r} 3x - 4y = 5 \\ (-) 4x - 4y = 0 \\ \hline (-) \quad (-) \quad (-) \\ -x \quad = 5 \end{array}$$

$$\therefore x = -5$$

Put  $x = -5$  in (1)

$$\begin{array}{r} 3(-5) - 4y = 5 \\ -15 - 4y = 5 \\ (-) -4y = 20 \\ y = -5 \end{array}$$

$$\therefore \boxed{x = \frac{5}{7} \text{ or } -5 \quad ; \quad y = \frac{-5}{7} \text{ or } -5}$$

10) Find  $z_1, z_2$  and  $\frac{z_1}{z_2}$

where  $z_1 = -1 + i\sqrt{3}$   
 $z_2 = 2 + i2\sqrt{3}$

Soln:  $z_1 z_2 = (-1 + i\sqrt{3})(2 + i2\sqrt{3})$   
 $= -1(2 + i2\sqrt{3}) + i\sqrt{3}(2 + i2\sqrt{3})$   
 $= -2 - i2\sqrt{3} + i2\sqrt{3} + i^2 6$   
 $= -2 + (-1)6$   
 $= -2 - 6$

$z_1 z_2 = -8$

$\frac{z_1}{z_2} = \frac{-1 + i\sqrt{3}}{2 + i2\sqrt{3}}$

$= \frac{-1 + i\sqrt{3}}{2 + i2\sqrt{3}} \times \frac{2 - i2\sqrt{3}}{2 - i2\sqrt{3}}$

$= \frac{(-1 + i\sqrt{3}) - 1(2 - i2\sqrt{3}) + i\sqrt{3}(2 - i2\sqrt{3})}{4 - i^2 12}$

$= \frac{-2 + i2\sqrt{3} + i2\sqrt{3} - i^2 6}{4 - (-1)12}$

$= \frac{-2 + i4\sqrt{3} - (-1)6}{4 + 12}$

$= \frac{-2 + i4\sqrt{3} + 6}{16}$

$= \frac{4 + i4\sqrt{3}}{16}$

$= \frac{1 + i\sqrt{3}}{4}$

$\frac{z_1}{z_2} = \frac{1 + i\sqrt{3}}{4}$

$z_1 = -1 + i\sqrt{3}$   
 $z_2 = 2 + i2\sqrt{3}$

ii) If  $z = 3 + i\sqrt{5}$ ; show that  $6z = z^2 + 14$

$$\begin{aligned} \text{L.H.S} &= 6z \\ &= 6(3 + i\sqrt{5}) \\ &= 18 + i6\sqrt{5} \end{aligned}$$

$$\begin{aligned} \text{R.H.S} &= z^2 + 14 \\ &= (3 + i\sqrt{5})^2 + 14 \\ &= 9 + 2(3)(i\sqrt{5}) + (i\sqrt{5})^2 + 14 \\ &= 9 + i6\sqrt{5} + i^2 5 + 14 \\ &= 23 + i6\sqrt{5} + (-1)5 \\ &= 23 - 5 + i6\sqrt{5} \\ &= 18 + i6\sqrt{5} \end{aligned}$$

$\therefore \text{L.H.S} = \text{R.H.S}$  ,,  
Hence proved.

12) Find the value of  $z^3 + z^2 - z + 22$  if  $z = \frac{5}{1-2i}$

$$\begin{aligned} \therefore z &= \frac{5}{1-2i} \times \frac{1+2i}{1+2i} \\ &= \frac{5+10i}{1-4i^2} \\ &= \frac{5+10i}{1-(-1)4} \\ &= \frac{5+10i}{1+4} \\ &= \frac{5+10i}{5} = \frac{1+i2}{1} \\ z &= 1+i2 \end{aligned}$$

$$\begin{aligned} \therefore z^3 &= (1+i2)^3 \\ &= 1 + (i2)^3 + 3(1)^2(i2) + 3(1)(i2)^2 \\ &= 1 + (i)^3 8 + i6 + (i)^2 12 \\ &= 1 + (i^2)(i) 8 + i6 + (-1) 12 \\ &= 1 + (-1)i 8 + i6 - 12 \\ &= -11 - i8 + i6 \\ z^3 &= -11 - i2 \end{aligned}$$

$$\begin{aligned} z^2 &= (1+i2)^2 \\ &= 1 + 2(1)(i2) + (i2)^2 \\ &= 1 + i4 + (i)^2 4 \\ &= 1 + i4 + (-1)4 \\ &= 1 + i4 - 4 \end{aligned}$$

$$z^2 = -3 + i4$$

$$\begin{aligned} \therefore z^3 + z &= (-5 + i) + (-5 + i) \\ &= -10 + 2i \end{aligned}$$

$$\begin{aligned} \therefore z^3 + z^2 - z + 22 &= (-11 - i2) + (-3 + i4) - (-5 + i) + 22 \\ &= -11 - i2 - 3 + i4 + 5 - i + 22 \\ &= 7 \end{aligned}$$

$$\begin{aligned} & \cancel{z^3 + z^2 - z + 22} \\ & \therefore \cancel{(-5 + i4) + (-3 + i4) - (1 + i2) + 22} \\ & \cancel{= -5 + i4 - 3 + i4 - 1 - i2 + 22} \\ & \cancel{= \boxed{13 + i6}} \end{aligned}$$

$$\begin{aligned} & \therefore z^3 + z^2 - z + 22 \\ & = (-11 - i2) + (-3 + i4) - (1 + i2) + 22 \\ & = -11 - i2 - 3 + i4 - 1 - i2 + 22 \\ & = \boxed{7} \end{aligned}$$

13) If  $z_1 = 4 - i5$ ;  $z_2 = 3 + i7$   
 find  $|3z_1 - 2z_2|$  &  $|z_1 z_2|$

Soln:

$$\begin{aligned} \text{let } z &= 3z_1 - 2z_2 \\ &= 3(4 - i5) - 2(3 + i7) \\ &= 12 - i15 - 6 - i14 \\ z &= 6 - i29 \\ x &= 6 ; y = -29 \end{aligned}$$

$$\begin{aligned} |z| &= \sqrt{x^2 + y^2} \\ &= \sqrt{6^2 + (-29)^2} \\ &= \sqrt{36 + 841} \end{aligned}$$

$$\begin{aligned} |z| &= \sqrt{877} \\ \therefore |3z_1 - 2z_2| &= \sqrt{877} \end{aligned}$$

$$\begin{aligned} \text{let } z &= z_1 z_2 \\ &= (4 - i5)(3 + i7) \\ &= 4(3 + i7) - i5(3 + i7) \\ &= 12 + i28 - i15 - i^2 35 \\ &= 12 + i13 + (-1)35 \\ &= 12 + i13 + 35 \end{aligned}$$

~~$$\begin{aligned} z &= 47 + i13 \\ x &= 47 ; y = 13 \\ |z| &= \sqrt{x^2 + y^2} \\ &= \sqrt{(47)^2 + (13)^2} \\ &= \sqrt{2209 + 169} \\ |z| &= \sqrt{2378} \\ |z_1 z_2| &= \sqrt{698} \end{aligned}$$~~

$$\begin{aligned} x &= 47 ; y = 13 \\ |z| &= \sqrt{x^2 + y^2} \\ &= \sqrt{(47)^2 + (13)^2} \\ &= \sqrt{2209 + 169} \\ |z| &= \sqrt{2378} \\ |z_1 z_2| &= \sqrt{2378} \end{aligned}$$

14) Express following in polar form

$$z = -2 - i2\sqrt{3}$$

$$x = -2 ; y = -2\sqrt{3}$$

$$r = \sqrt{x^2 + y^2}$$

$$= \sqrt{(-2)^2 + (-2\sqrt{3})^2}$$

$$= \sqrt{4 + 12}$$

$$r = \sqrt{16} \quad r = 4$$

$$\theta = \tan^{-1} \left( \frac{y}{x} \right)$$

$$= \tan^{-1} \frac{-2\sqrt{3}}{-2}$$

$$= \tan^{-1} \sqrt{3}$$

$$\theta = \frac{\pi}{3}$$

∴ Polar form =

$$z = r(\cos \theta + i \sin \theta)$$

$$= \boxed{\sqrt{16} \left( \cos \frac{\pi}{3} + i \sin \frac{\pi}{3} \right)}$$

$$15) z = \frac{-1}{2} - i \frac{\sqrt{3}}{2}$$

$$x = \frac{-1}{2} \quad ; \quad y = -\frac{\sqrt{3}}{2}$$

$$\begin{aligned} r &= \sqrt{x^2 + y^2} \\ &= \sqrt{\left(\frac{-1}{2}\right)^2 + \left(-\frac{\sqrt{3}}{2}\right)^2} \\ &= \sqrt{\frac{1}{4} + \frac{3}{4}} \\ &= \sqrt{\frac{4}{4}} \end{aligned}$$

$$r = 1$$

∴ Polar form,

$$\begin{aligned} z &= r(\cos \theta + i \sin \theta) \\ &= 1 \left( \cos \frac{\pi}{3} + i \sin \frac{\pi}{3} \right) \\ &= \boxed{\cos \frac{\pi}{3} + i \sin \frac{\pi}{3}} \end{aligned}$$

$$16) z = \frac{(1+i)(2+i)}{3-i}$$

$$\therefore z = \frac{(1+i)(2+i)}{3-i} \times \frac{3+i}{3+i}$$

$$= \frac{[1(2+i) + i(2+i)] \times 3+i}{9-i^2}$$

$$= \frac{(2+i+2+i^2)(3+i)}{9+1}$$

$$= \frac{(2+i+2-1)(3+i)}{10}$$

$$= \frac{(1+i)(3+i)}{10}$$

$$= \frac{1(3+i) + i(3+i)}{10}$$

$$= \frac{3+i+i^2+3i}{10}$$

$$= \frac{3+i+3i-1}{10}$$

$$= \frac{3-1+i+3i}{10}$$

$$= \frac{i+3i}{10}$$

$$z = i$$

$$x = 0; y = 1$$

$$r = \sqrt{x^2 + y^2}$$

$$r = \sqrt{0^2 + 1^2} = \sqrt{1}$$

$$\therefore r = 1$$

$$\theta = \tan^{-1} \left( \frac{y}{x} \right)$$

$$= \tan^{-1} \left( \frac{1}{0} \right) = \tan^{-1} \infty$$

$$\theta = \frac{\pi}{2}$$

$\therefore$  Polar form

$$z = r (\cos \theta + i \sin \theta)$$

$$= 1 \left( \cos \frac{\pi}{2} + i \sin \frac{\pi}{2} \right)$$

$$z = \cos \frac{\pi}{2} + i \sin \frac{\pi}{2}$$

$$12) z = \frac{2+i6\sqrt{3}}{5+i\sqrt{3}}$$

$$\therefore z = \frac{2+i6\sqrt{3}}{5+i\sqrt{3}} \times \frac{5-i\sqrt{3}}{5-i\sqrt{3}}$$

$$= \frac{2(5-i\sqrt{3}) + i6\sqrt{3}(5-i\sqrt{3})}{25 - i^2 3}$$

$$= \frac{10 - i2\sqrt{3} + i30\sqrt{3} - i^2 18}{25 - (-1)3}$$

$$= \frac{10 + i28\sqrt{3} + 18}{28}$$

$$= \frac{28 + i28\sqrt{3}}{28}$$

$$= \frac{28(1+i\sqrt{3})}{28}$$

$$z = 1+i\sqrt{3}$$

$$x=1; y=\sqrt{3}$$

$$r = \sqrt{x^2+y^2}$$

$$= \sqrt{1^2+(\sqrt{3})^2}$$

$$= \sqrt{1+3} = \sqrt{4}$$

$$r = 2$$

$$\theta = \tan^{-1}\left(\frac{y}{x}\right)$$

$$= \tan^{-1}\left(\frac{\sqrt{3}}{1}\right)$$

$$= \tan^{-1}\sqrt{3}$$

$$\theta = \frac{\pi}{3}$$

$\therefore$  Polar form

$$z = r(\cos\theta + i\sin\theta)$$

$$z = 2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$$

$$\frac{\sin\pi}{2}$$

$$18) z = 3\sqrt{2}(-1+i) \\ = -3\sqrt{2} + i3\sqrt{2}$$

$$x = -3\sqrt{2} ; y = 3\sqrt{2}$$

$$r = \sqrt{x^2 + y^2} \\ = \sqrt{(-3\sqrt{2})^2 + (3\sqrt{2})^2} \\ = \sqrt{18 + 18} \\ = \sqrt{36}$$

$$r = 6$$

$$\theta = \tan^{-1} \frac{y}{x} \\ = \tan^{-1} \left( \frac{3\sqrt{2}}{-3\sqrt{2}} \right) \\ = \tan^{-1}(-1) \\ \theta = -\frac{\pi}{4}$$

$\therefore$  Polar form,

$$z = r(\cos \theta + i \sin \theta) \\ = 6 \left( \cos \left( -\frac{\pi}{4} \right) + i \sin \left( -\frac{\pi}{4} \right) \right)$$

$$z = 6 \left( \cos \left( \frac{\pi}{4} \right) - i \sin \left( \frac{\pi}{4} \right) \right)$$

19) If find

soln: let  $z =$

(9) If  $z_1 = 1 + i2$  ;  $z_2 = 2 - i3$   
Find  $\left| \frac{z_1}{z_2} \right|$

soln:

let  $z = \frac{z_1}{z_2}$

$$= \frac{1+i2}{2-i3}$$

$$= \frac{1+i2}{2-i3} \times \frac{2+i3}{2+i3}$$

$$= \frac{1(2+i3) + i2(2+i3)}{4 - i^2 9}$$

$$= \frac{2+i3 + i4 + i^2 6}{4 - (-9)}$$

$$= \frac{2+i7 + (-1)6}{4+9}$$

$$= \frac{-4+i7}{13}$$

$$\left\{ z = \frac{-4}{13} + i \frac{7}{13} \right\} \quad x = \frac{-4}{13}, \quad y = \frac{7}{13}$$

$$\therefore |z| = \sqrt{x^2 + y^2}$$

$$= \sqrt{\left(\frac{-4}{13}\right)^2 + \left(\frac{7}{13}\right)^2}$$

$$= \sqrt{\frac{16 + 49}{169}}$$

$$= \sqrt{\frac{65}{169}}$$

$$|z| = \frac{\sqrt{65}}{13}$$

$$|z| = \frac{\sqrt{65}}{13}$$

$$\left| \frac{z_1}{z_2} \right| = \frac{\sqrt{65}}{13}$$

$$\therefore \left| \frac{z_1}{z_2} \right| = \frac{\sqrt{65}}{13}$$

Q20) show that  $\frac{\sqrt{3} + i}{2}$  is a cube root of  $i$ .

Soln: To prove:  $\sqrt[3]{i} = \frac{\sqrt{3} + i}{2}$

T.P.T:  $i^{1/3} = \frac{\sqrt{3} + i}{2}$

T.P.T cubing on both sides,  
 $i = \left(\frac{\sqrt{3} + i}{2}\right)^3$

L.H.S =  $\left(\frac{\sqrt{3} + i}{2}\right)^3$

=  $\left(\frac{\sqrt{3}}{2}\right)^3 + \left(\frac{i}{2}\right)^3 + 3\left(\frac{\sqrt{3}}{2}\right)^2\left(\frac{i}{2}\right) + 3\left(\frac{\sqrt{3}}{2}\right)\left(\frac{i}{2}\right)^2$

=  $\frac{3\sqrt{3}}{8} + \frac{i^3}{8} + \frac{i \cdot 9}{8} + \frac{i^2 \cdot 3\sqrt{3}}{8}$

=  $\frac{3\sqrt{3}}{8} + \frac{(-1)}{8} + \frac{i \cdot 9}{8} + \frac{(-1) \cdot 3\sqrt{3}}{8}$

=  $\frac{3\sqrt{3}}{8} - \frac{1}{8} + \frac{i \cdot 9}{8} - \frac{3\sqrt{3}}{8}$

=  $\frac{i \cdot 9}{8}$

=  $i$

=  $i$

= L.H.S

Hence proved