Tutorial-2

Q1) $$\frac{(\cos 2\theta + i\sin 2\theta)^{3/2}}{(\cos 3\theta - i\sin 3\theta)^2} \cdot \frac{(\cos\theta - i\sin\theta)^8}{(\cos 5\theta - i\sin 5\theta)^{2/5}}$$

$$\therefore (\cos 2\theta + i\sin 2\theta)^{3/2}$$
$$= (\cos\theta + i\sin\theta)^{2\times 3/2}$$
$$= (\cos\theta + i\sin\theta)^3$$

$$(\cos\theta - i\sin\theta)^8$$
$$= (\cos\theta + i\sin\theta)^{-1\times 8}$$
$$= (\cos\theta + i\sin\theta)^{-8}$$

$$(\cos 3\theta - i\sin 3\theta)^2$$
$$= (\cos\theta + i\sin\theta)^{-3\times 2}$$
$$= (\cos\theta + i\sin\theta)^{-6}$$

$$(\cos 5\theta - i\sin 5\theta)^{2/5}$$
$$= (\cos\theta + i\sin\theta)^{-\cancel{5}\times 2/\cancel{5}}$$
$$= (\cos\theta + i\sin\theta)^{-2}$$

Using De'movier's thm.

$\therefore$ Given exp.

$$= \frac{(\cos\theta + i\sin\theta)^3 \times (\cos\theta + i\sin\theta)^{-8}}{(\cos\theta + i\sin\theta)^{-6} \times (\cos\theta + i\sin\theta)^{-2}}$$

$$= (\cos\theta + i\sin\theta)^{3+(-8)-(-6-2)}$$

$$= (\cos\theta + i\sin\theta)^3$$

$$= \boxed{\cos 3\theta + i\sin 3\theta}$$

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Q2) $\dfrac{(\cos 3\theta + i\sin 3\theta)^4 \; (\cos 4\theta - i\sin 4\theta)^5}{(\cos 4\theta + i\sin 4\theta)^3 \; (\cos 5\theta + i\sin 5\theta)^{-4}}$

$(\cos 3\theta + i\sin 3\theta)^4$
$= (\cos\theta + i\sin\theta)^{3\times 4}$
$= (\cos\theta + i\sin\theta)^{12}$

$(\cos 4\theta - i\sin 4\theta)^5$
$= (\cos\theta + i\sin\theta)^{-4\times 5}$
$= (\cos\theta + i\sin\theta)^{-20}$

$(\cos 4\theta + i\sin 4\theta)^3$
$= (\cos\theta + i\sin\theta)^{4\times 3}$
$= (\cos\theta + i\sin\theta)^{12}$

$(\cos 5\theta + i\sin 5\theta)^{-4}$
$= (\cos\theta + i\sin\theta)^{5\times -4}$
$= (\cos\theta + i\sin\theta)^{-20}$

Using De'mouier's thm.

$\therefore$ Given exp:

$$= \frac{(\cos\theta + i\sin\theta)^{12} \times (\cos\theta + i\sin\theta)^{-20}}{(\cos\theta + i\sin\theta)^{12} \times (\cos\theta + i\sin\theta)^{-20}}$$

$= (\cos\theta + i\sin\theta)^{12+(-20)-12-(+20)}$
$= (\cos\theta + i\sin\theta)^{\cancel{12}-\cancel{20}-\cancel{12}+\cancel{20}}$
$= (\cos\theta + i\sin\theta)^0$
$= \cos(0) + i\sin(0)$
$= 1 + i(0)$
$= \boxed{1}$

Q3) $\dfrac{(\cos 2\theta - i\sin 2\theta)^5 (\cos 3\theta + i\sin 3\theta)^6}{(\cos 4\theta + i\sin 4\theta)^7 (\cos\theta - i\sin\theta)^3}$

$$\left.\begin{aligned}
&(\cos 2\theta - i\sin 2\theta)^5 \\
&= (\cos\theta + i\sin\theta)^{-2\times 5} \\
&= (\cos\theta + i\sin\theta)^{-10} \\[1em]
&(\cos 3\theta + i\sin 3\theta)^6 \\
&= (\cos\theta + i\sin\theta)^{3\times 6} \\
&= (\cos\theta + i\sin\theta)^{18} \\[1em]
&(\cos 4\theta + i\sin 4\theta)^7 \\
&= (\cos\theta + i\sin\theta)^{4\times 7} \\
&= (\cos\theta + i\sin\theta)^{28} \\[1em]
&(\cos\theta - i\sin\theta)^3 \\
&(\cos\theta + i\sin\theta)^{-1\times 3} \\
&(\cos\theta + i\sin\theta)^{-3}
\end{aligned}\right\} \text{Using De'movier's thm.}$$

$\therefore$ Given exp,

$$= \frac{(\cos\theta + i\sin\theta)^{-10} \times (\cos\theta + i\sin\theta)^{18}}{(\cos\theta + i\sin\theta)^{28} \times (\cos\theta + i\sin\theta)^{-3}}$$

$$= \frac{(\cos\theta + i\sin\theta)^{-10+18 \cancel{-(28-3)}}}{(\cos\theta + i\sin\theta)^{28+(-3)}}$$

$$= \frac{(\cos\theta + i\sin\theta)^{8}}{(\cos\theta + i\sin\theta)^{25}} = (\cos\theta + i\sin\theta)^{8-25}$$

$$= (\cos\theta + i\sin\theta)^{-17} = (\cos(-17\theta) + i\sin(-17\theta))$$

$$= \boxed{\cos 17\theta - i\sin 17\theta}$$

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Q4) $$\frac{(\cos 5\theta - i\sin 5\theta)^2 (\cos 7\theta + i\sin 7\theta)^{-3}}{(\cos 4\theta - i\sin 4\theta)^9 (\cos\theta + i\sin\theta)^5}$$

$$(\cos 5\theta - i\sin 5\theta)^2$$
$$= (\cos\theta + i\sin\theta)^{-5\times 2}$$
$$= (\cos\theta + i\sin\theta)^{-10}$$

$$(\cos 7\theta + i\sin 7\theta)^{-3}$$
$$= (\cos\theta + i\sin\theta)^{7\times -3}$$
$$= (\cos\theta + i\sin\theta)^{-21}$$

$$(\cos 4\theta - i\sin 4\theta)^9$$
$$= (\cos\theta + i\sin\theta)^{-4\times 9}$$
$$= (\cos\theta + i\sin\theta)^{-36}$$

$$(\cos\theta + i\sin\theta)^5$$

Using De'mouier's thm.

$\therefore$ Given exp,

$$= \frac{(\cos\theta + i\sin\theta)^{-10} \times (\cos\theta + i\sin\theta)^{-21}}{(\cos\theta + i\sin\theta)^{-36} \times (\cos\theta + i\sin\theta)^5}$$

$$= (\cos\theta + i\sin\theta)^{-10-21+36-5}$$

$$= (\cos\theta + i\sin\theta)^0$$

Q5) $\dfrac{(\cos\theta + i\sin\theta)^4}{(\sin\theta + i\cos\theta)^5}$

$\therefore (\cos\theta + i\sin\theta)^4$
$= e^{i\theta \times 4}$
$= e^{i4\theta}$

$(\sin\theta + i\cos\theta)^5$
$= \left(\cos\left(\frac{\pi}{2}-\theta\right) + i\sin\left(\frac{\pi}{2}-\theta\right)\right)^5$
$= e^{i\left(\frac{\pi}{2}-\theta\right)\times 5}$
$= e^{i\left(\frac{5\pi}{2}-5\theta\right)}$

Using Euler's formula.

$\therefore$ Given exp,
$= \dfrac{e^{i4\theta}}{e^{i\left(\frac{5\pi}{2}-5\theta\right)}}$
$= e^{i4\theta - i\left(\frac{5\pi}{2}-5\theta\right)}$
$= e^{i\left(9\theta - \frac{5\pi}{2}\right)}$
$= e^{i9\theta} \cdot e^{-i\frac{5\pi}{2}}$

$\therefore e^{i9\theta} = \cos 9\theta + i\sin 9\theta$

$e^{-i\frac{5\pi}{2}} = \cos\frac{5\pi}{2} - i\sin\frac{5\pi}{2}$
$= -i$

$\therefore -i \times e^{i9\theta} = -i(\cos 9\theta + i\sin 9\theta)$
$= -i\cos 9\theta - i^2\sin 9\theta$
$= \boxed{\sin 9\theta - i\cos 9\theta}$

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To prove :

Q6) $\left(\sin\frac{\pi}{6} + i\cos\frac{\pi}{6}\right)^7 = \frac{1+i\sqrt{3}}{2}$

L.H.S. $= \left(\sin\frac{\pi}{6} + i\cos\frac{\pi}{6}\right)^7$

$= \left(\cos\left(\frac{\pi}{2}-\frac{\pi}{6}\right) + i\sin\left(\frac{\pi}{2}-\frac{\pi}{6}\right)\right)^7$

$= \left(\cos\left(\frac{3\pi-\pi}{6}\right) + i\sin\left(\frac{3\pi-\pi}{6}\right)\right)^7$

$= \left(\cos\left(\frac{2\pi}{6}\right) + i\sin\left(\frac{2\pi}{6}\right)\right)^7$

$= \left(\cos\left(\frac{\pi}{3}\right) + i\sin\left(\frac{\pi}{3}\right)\right)^7$

$= \cos\left(\frac{7\pi}{3}\right) + i\sin\left(\frac{7\pi}{3}\right)$ [Using De'movier's thm]

$= \frac{1}{2} + i\frac{\sqrt{3}}{2}$

$= \boxed{\frac{1+i\sqrt{3}}{2}}$

= R.H.S

"Hence proved

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Q7) If $z_1 = 2 + i3$ find $\sqrt{\frac{z_1}{z_2}}$

$z_2 = 3 - i2$

Soln :. $\frac{z_1}{z_2} = \frac{2+i3}{3-i2}$

$= \frac{2+i3}{3-i2} \times \frac{3+i2}{3+i2}$

$= \frac{2(3+i2) + i3(3+i2)}{9 - (i^2)4}$

$= \frac{6 + i4 + i9 + i^2 6}{9 - (-1)4}$

$= \frac{6 + i13 + (-1)6}{13}$

$= \frac{\cancel{0}\, i13}{13}$

$\frac{z_1}{z_2} = i$

$\therefore \sqrt{\frac{z_1}{z_2}} = \sqrt{i} = i^{1/2}$

$x = 0 \;;\; y = 1$

$r = \sqrt{x^2+y^2} = \sqrt{0^2+1^2} = \sqrt{1} = 1$

$\theta = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{1}{0}\right) = \tan^{-1}(\infty) = \frac{\pi}{2}$

$\therefore$ polar form,

$z = r(\cos\theta + i\sin\theta)$

$\therefore i = 1\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)$

$i^{1/2} = \left(\cos\left(\frac{\pi}{2}\right) + i\sin\left(\frac{\pi}{2}\right)\right)^{1/2}$

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$$\sqrt{i} = \cos\frac{\pi}{4} + i\sin\frac{\pi}{4}$$ [using De'moivier's thm]

$$= \frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}}$$

$$\sqrt{i} = \frac{1+i}{\sqrt{2}}$$

$$\therefore \boxed{\sqrt{\frac{z_1}{z_2}} = \frac{1+i}{\sqrt{2}}}$$

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find values of:

Q8) $(1+i)^8 + (1-i)^8$

cosider, $z_1 = (1+i)$

$x = 1$, $y = 1$

$r = \sqrt{x^2+y^2} = \sqrt{1^2+(1)^2} = \sqrt{2}$

$\theta = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{1}{1}\right) = \tan^{-1}(1) = \frac{\pi}{4}$

$\therefore$ Polar form,

$z_1 = r(\cos\theta + i\sin\theta)$

$= \sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$

cosider, $z_2 = (1-i)$

$x = 1$, $y = -1$

$r = \sqrt{x^2+y^2} = \sqrt{1^2+1^2} = \sqrt{2}$

$\theta = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{-1}{1}\right) = \tan\left(-\frac{\pi}{4}\right)$

$\therefore$ Polar form,

$z_2 = r(\cos\theta + i\sin\theta)$

$= \sqrt{2}\left(\cos\left(-\frac{\pi}{4}\right) + i\sin\left(-\frac{\pi}{4}\right)\right)$

$= \sqrt{2}\left(\cos\frac{\pi}{4} - i\sin\frac{\pi}{4}\right)$

$\therefore (1+i)^8 + (1-i)^8$

$= \left(\sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)\right)^8 + \left(\sqrt{2}\left(\cos\frac{\pi}{4} - i\sin\frac{\pi}{4}\right)\right)^8$

$= (\sqrt{2})^8\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)^8 + (\sqrt{2})^8\left(\cos\frac{\pi}{4} - i\sin\frac{\pi}{4}\right)^8$

$= (\sqrt{2})^8 ($

$= (\sqrt{2})^8 ($

$= 16\ (2$

$= 16 \times 2($

$= \boxed{32}$ //

Q9) $(1+i\sqrt{3})$

consider, $z_1 = ($

x

$r = \sqrt{x}$

$\theta =$

$\therefore$ Polar

$z_1 =$

$z_1 =$

conside z_2

x

$r =$

$\theta =$

$\therefore$ Polar

$z_2 =$

z_2

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$= (\sqrt{2})^8 \left(\cos \frac{8\pi}{4} + i \sin \frac{8\pi}{4}\right) + (\sqrt{2})^8 \left(\cos \frac{8\pi}{4} - i \sin \frac{8\pi}{4}\right)$

$= (\sqrt{2})^8 \left(\cos 2\pi + \cancel{i \sin 2\pi} + \cos 2\pi - \cancel{i \sin 2\pi}\right)$

$= 16 (2 \cos 2\pi)$

$= 16 \times 2(1)$

$= \boxed{32}$ // Using De'movier's thm.

Q9) $(1+i\sqrt{3})^6 + (1-i\sqrt{3})^6$

consider, $z_1 = (1+i\sqrt{3})$

$x = 1, \quad y = \sqrt{3}$

$r = \sqrt{x^2+y^2} = \sqrt{(1)^2+(\sqrt{3})^2} = \sqrt{4} = 2$

$\theta = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{\sqrt{3}}{1}\right) = \tan^{-1}\sqrt{3} = \frac{\pi}{3}$

$\therefore$ Polar form,

$z_1 = r(\cos\theta + i \sin\theta)$

$z_1 = 2\left(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3}\right)$

consider $z_2 = (1-i\sqrt{3})$

$x = 1, \quad y = -\sqrt{3}$

$r = \sqrt{x^2+y^2} = \sqrt{(1)^2+(-\sqrt{3})^2} = \sqrt{4} = 2$

$\theta = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{-\sqrt{3}}{1}\right) = \tan^{-1}(-\sqrt{3}) = -\frac{\pi}{3}$

$\therefore$ Polar form,

$z_2 = r(\cos\theta + i \sin\theta)$

$= 2\left(\cos\left(-\frac{\pi}{3}\right) + i \sin\left(-\frac{\pi}{3}\right)\right)$

$z_2 = 2\left(\cos \frac{\pi}{3} - i \sin \frac{\pi}{3}\right)$

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$\therefore (1+\sqrt{3})^6 + (1-\sqrt{3})^6$

$= \left(2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)\right)^6 + \left(2\left(\cos\frac{\pi}{3} - i\sin\frac{\pi}{3}\right)\right)^6$

$= (2)^6\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)^6 + (2)^6\left(\cos\frac{\pi}{3} - i\sin\frac{\pi}{3}\right)^6$

$= (2)^6\left(\cos\frac{6\pi}{3} + i\sin\frac{6\pi}{3}\right) + (2)^6\left(\cos\frac{6\pi}{3} - i\sin\frac{6\pi}{3}\right)$

$= (2)^6\left(\cos 2\pi + \cancel{i\sin 2\pi} + \cos 2\pi - \cancel{i\sin 2\pi}\right)$

$= 64 \times (2\cos 2\pi)$

$= 64 \times 2(1)$

$= \boxed{128}$ // Using De'movier's thm.

10) $(-2\sqrt{3} + i2)^{12} + (-2\sqrt{3} - i2)^{12}$

$z_1 = (-2\sqrt{3} + i2)$

$x = -2\sqrt{3}$, $y = 2$

$r = \sqrt{x^2+y^2} = \sqrt{(-2\sqrt{3})^2 + (2)^2} = \sqrt{12+4} = \sqrt{16} = 4$

$\theta = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{2}{-2\sqrt{3}}\right) = \tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) = -\frac{\pi}{6}$

$\therefore$ Polar form, of

$z_1 = r(\cos\theta + i\sin\theta)$

$= 4\left(\cos\left(-\frac{\pi}{6}\right) + i\sin\left(-\frac{\pi}{6}\right)\right)$

$z_1 = 4\left(\cos\frac{\pi}{6} - i\sin\frac{\pi}{6}\right)$

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consider $z_2 = (-2\sqrt{3} - i2)$

$x = -2\sqrt{3}$; $y = -2$

$r = \sqrt{x^2+y^2} = \sqrt{(-2\sqrt{3})^2 + (-2)^2} = \sqrt{12+4} = \sqrt{16} = 4$

$\theta = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{-2}{-2\sqrt{3}}\right) = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{6}$

$\therefore$ Polar form of

$z_2 = r(\cos\theta + i\sin\theta)$

$z_2 = 4\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$

$\therefore$ $(-2\sqrt{3}+i2)^{12} + (-2\sqrt{3}-i2)^{12}$

$= \left(4\left(\cos\frac{\pi}{6} - i\sin\frac{\pi}{6}\right)\right)^{12} + \left(4\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)\right)^{12}$

$= (4)^{12}\left(\cos\frac{\pi}{6} - i\sin\frac{\pi}{6}\right)^{12} + (4)^{12}\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)^{12}$

$= (4)^{12}\left(\cos\frac{12\pi}{6} - i\sin\frac{12\pi}{6}\right) + (4)^{12}\left(\cos\frac{12\pi}{6} + i\sin\frac{12\pi}{6}\right)$

$= (4)^{12}\left(\cos 2\pi - \cancel{i\sin 2\pi} + \cos 2\pi + \cancel{i\sin 2\pi}\right)$

$= (4)^{12} \times (2\cos 2\pi)$

$= 16777216 \times 2(1)$

$= \boxed{33554432}$ // Using Demovier's thm.

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Convert to $x+iy$ form :

Q11) $2e^{i(\pi/3)}$

Let $z = 2e^{i(\pi/3)}$

$= 2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$ Using Euler's form

$= 2\left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)$

$\boxed{z = 1 + i\sqrt{3}}$

Q12) $\sqrt{3}\, e^{-i(\pi/6)}$

Let $z = \sqrt{3}\, e^{-i(\pi/6)}$

$= \sqrt{3}\left(\cos\left(\frac{\pi}{6}\right) - i\sin\left(\frac{\pi}{6}\right)\right)$ Using Euler's formula

$= \sqrt{3}\left(\frac{\sqrt{3}}{2} - i\frac{1}{2}\right)$

$\boxed{z = \frac{3}{2} - i\frac{\sqrt{3}}{2}}$

Q13) $6e^{-i(2\pi/3)}$

Let $z = 6e^{-i(2\pi/3)}$

$= 6\left(\cos\left(\frac{2\pi}{3}\right) - i\sin\left(\frac{2\pi}{3}\right)\right)$ Using Euler's form

$= 6\left(-\frac{1}{2} - i\frac{\sqrt{3}}{2}\right)$

$\boxed{z = -3 - i\,3\sqrt{3}}$

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