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B014
CSE-B

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Tutorial-3

Q1) Find the magnitude of vectors

a] $2\hat{i} + 2\hat{j} - \hat{k}$

Let $\bar{a} = 2\hat{i} + 2\hat{j} - \hat{k}$

$$\therefore |\bar{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$$
$$= \sqrt{(2)^2 + (2)^2 + (-1)^2}$$
$$= \sqrt{4+4+1}$$
$$= \sqrt{9}$$
$$\boxed{|\bar{a}| = 3}$$

b] $2\hat{i} - 3\hat{j} + \hat{k}$

Let $\bar{a} = 2\hat{i} - 3\hat{j} + \hat{k}$

$$\therefore |\bar{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$$
$$= \sqrt{(2)^2 + (-3)^2 + (1)^2}$$
$$= \sqrt{4+9+1}$$
$$\boxed{|\bar{a}| = \sqrt{14}}$$

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Q2) $\bar{p}_1 = 5\hat{i} + 2\hat{j} + 3\hat{k}$; find the magnitude of

$\bar{p}_2 = 2\hat{i} + \hat{j} - \hat{k}$

a) $\bar{p}_1 + \bar{p}_2$

b) $2\bar{p}_1 + 3\bar{p}_2$

a) ∴ Let $\bar{a} = \bar{p}_1 + \bar{p}_2$

$= 5\hat{i} + 2\hat{j} + 3\hat{k} + 2\hat{i} + \hat{j} - \hat{k}$

$= (5+2)\hat{i} + (2+1)\hat{j} + (3-1)\hat{k}$

$\boxed{\bar{a} = 7\hat{i} + 3\hat{j} + 2\hat{k}}$

b) $2\bar{p}_1 = 2(5\hat{i} + 2\hat{j} + 3\hat{k})$

$= (10\hat{i} + 4\hat{j} + 6\hat{k})$

$3\bar{p}_2 = 3(2\hat{i} + \hat{j} - \hat{k})$

$= (6\hat{i} + 3\hat{j} - 3\hat{k})$

∴ $2\bar{p}_1 + 3\bar{p}_2 = 10\hat{i} + 4\hat{j} + 6\hat{k} + 6\hat{i} + 3\hat{j} - 3\hat{k}$

$= (10+6)\hat{i} + (4+3)\hat{j} + (6-3)\hat{k}$

$\boxed{2\bar{p}_1 + 3\bar{p}_2 = 16\hat{i} + 7\hat{j} + 3\hat{k}}$

a) $|\bar{p}_1 + \bar{p}_2| = \sqrt{(7)^2 + (3)^2 + (2)^2}$

$= \sqrt{49 + 9 + 4}$

$\boxed{|\bar{p}_1 + \bar{p}_2| = \sqrt{62}}$

b) $|2\bar{p}_1 + 3\bar{p}_2| = \sqrt{(16)^2 + (7)^2 + (3)^2}$

$= \sqrt{256 + 49 + 9}$

$\boxed{|2\bar{p}_1 + 3\bar{p}_2| = \sqrt{314}}$

Q3) If A(1,2,3) ...
express vector ...

$\bar{OA} = +i + 2j$ ...

$\bar{OB} = -1i + -3$ ...

$\bar{OC} = 4i, -j$ ...

∴ $\bar{AB} = \bar{OB} -$ ...

$= (-i-$ ...

$= -i$ ...

$\bar{AB} = -$ ...

$\bar{BC} = \bar{OC}$ ...

$= (4i$ ...

$= (4$ ...

$\bar{BC} = 5$ ...

$\bar{CA} =$ ...

$= (i$ ...

$= i$ ...

$\bar{CA} = -$ ...

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Q3) If $A(1,2,3)$; $B(-1,0,-3)$; $C(4,-1,3)$
express vectors $\overline{AB}$, $\overline{BC}$ and $\overline{CA}$ in $\hat{i}, \hat{j}, \hat{k}$

$\overline{OA} = +i + 2j + 3k$
$\overline{OB} = -1i + -3k$
$\overline{OC} = 4i - j + 3k$

$\therefore \overline{AB} = \overline{OB} - \overline{OA}$
$= (-i - 3k) - (i + 2j + 3k)$
$= -i - 3k - i - 2j - 3k$
$\boxed{\overline{AB} = -2i - 2j - 6k}$

$\overline{BC} = \overline{OC} - \overline{OB}$
$= (4i - j + 3k) - (-i - 3k)$
$= (4+1)i + (-1+0)j + (3+3)k$
$\boxed{\overline{BC} = 5i - j + 6k}$

$\overline{CA} = \overline{OA} - \overline{OC}$
$= (i + 2j + 3k) - (4i - j + 3k)$
$= i + 2j + 3k - 4i + j - 3k$
$\boxed{\overline{CA} = -3i + 3j}$

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Q4) Given 3 vectors $\bar{a}, \bar{b}, \bar{c}$ such that,

$$7\bar{a} = 2i + 3j + 6k$$
$$7\bar{b} = 3i - 6j + 2k$$
$$7\bar{c} = 6i + 2j + 3k$$

Show that $\bar{a}, \bar{b}, \bar{c}$ are each of unit length and prove $\bar{a}$ & $\bar{b}$ are $\perp$.

Soln:

Divide ~~thong~~ throughout by 7,

$$\therefore \bar{a} = \frac{2}{7}i + \frac{3}{7}j + \frac{6}{7}k$$

$$\bar{b} = \frac{3}{7}i - \frac{6}{7}j + \frac{2}{7}k$$

$$\bar{c} = \frac{6}{7}i + \frac{2}{7}j + \frac{3}{7}k$$

$$\therefore |\bar{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$$
$$= \sqrt{\left(\frac{2}{7}\right)^2 + \left(\frac{3}{7}\right)^2 + \left(\frac{6}{7}\right)^2}$$
$$= \sqrt{\frac{4}{49} + \frac{9}{49} + \frac{36}{49}} = \sqrt{\frac{49}{49}}$$
$$= \sqrt{1}$$
$$|\bar{a}| = 1$$

$$|\bar{b}| = \sqrt{b_1^2 + b_2^2 + b_3^2}$$
$$= \sqrt{\left(\frac{3}{7}\right)^2 + \left(\frac{-6}{7}\right)^2 + \left(\frac{2}{7}\right)^2}$$
$$= \sqrt{\frac{9}{49} + \frac{36}{49} + \frac{4}{49}}$$
$$= \sqrt{\frac{49}{49}}$$
$$= \sqrt{1}$$

$|\bar{b}| = 1$

$|\bar{c}| = \sqrt{c_1^2 + c\ldots}$

$= \sqrt{\left(\frac{6}{7}\right)^2 \ldots}$

$= \sqrt{\frac{36}{49} \ldots}$

$= \sqrt{\frac{49}{49}}$

$= \sqrt{1}$

$|\bar{c}| = 1$

$\therefore$ ~~$|\bar{a}|, |\bar{b}|, \ldots$~~

$|\bar{a}| = 1$; $|\bar{b} \ldots$

$\therefore \bar{a}, \bar{b}, \bar{c} \ldots$

$\therefore \bar{a} \cdot \bar{b} = \ldots$

$\boxed{\bar{a} \cdot \bar{b} = \ldots}$

$\because \bar{a} \cdot \bar{b} = 0, \ldots$

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$|\bar{b}| = 1$

$$|\bar{c}| = \sqrt{c_1^2 + c_2^2 + c_3^2}$$
$$= \sqrt{\left(\frac{6}{7}\right)^2 + \left(\frac{2}{7}\right)^2 + \left(\frac{3}{7}\right)^2}$$
$$= \sqrt{\frac{36}{49} + \frac{4}{49} + \frac{9}{49}}$$
$$= \sqrt{\frac{49}{49}}$$
$$= \sqrt{1}$$
$$|\bar{c}| = 1$$

$\therefore$ ~~$|\bar{a}|, |\bar{b}|, |\bar{c}|$~~

$|\bar{a}| = 1$; $|\bar{b}| = 1$; $|\bar{c}| = 1$

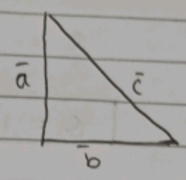
$\therefore$ $\bar{a}, \bar{b}, \bar{c}$ are all unit vectors.

$$\therefore \quad \bar{a}\cdot\bar{b} = (a_1 i + a_2 j + a_3 k)\cdot(b_1 i + b_2 j + b_3 k)$$
$$= \left(\frac{2}{7} i + \frac{3}{7} j + \frac{6}{7} k\right)\cdot\left(\frac{3}{7} i - \frac{6}{7} j + \frac{2}{7} k\right)$$
$$= \left(\frac{2}{7}\right)\left(\frac{3}{7}\right) + \left(\frac{3}{7}\right)\left(\frac{-6}{7}\right) + \left(\frac{6}{7}\right)\left(\frac{2}{7}\right)$$
$$= \frac{6}{49} - \frac{18}{49} + \frac{12}{49}$$
$$= \frac{18}{49} - \frac{18}{49}$$
$$\boxed{\bar{a}\cdot\bar{b} = 0}$$

$\because$ $\bar{a}\cdot\bar{b} = 0$, then $\bar{a} \perp \bar{b}$

Q5) Show that the vectors $2i - j + k$, $i - 3j - 5k$ and $3i - 4j - 4k$ form the sides of right angled triangle. Also find the remaining angles.

Let $\bar{a} = 2i - j + k$
$\bar{b} = i - 3j - 5k$
$\bar{c} = 3i - 4j - 4k$



$$\bar{a}\cdot\bar{b} = (2i - j + k)\cdot(i - 3j - 5k)$$
$$= (2)(1) + (-1)(-3) + (1)(-5)$$
$$= 2 + 3 - 5$$
$$\boxed{\bar{a}\cdot\bar{b} = 0}$$

$\because$ $\bar{a}\cdot\bar{b} = 0$; $\cancel{\bar{a}\perp\bar{b}}$ $\bar{a}$ is perpendicular to $\bar{b}$

$\therefore$ ~~they~~ $\bar{a}, \bar{b}, \bar{c}$ are sides of right angle triangle

Remaining angles,

$$|\bar{a}| = \sqrt{(2)^2 + (-1)^2 + (1)^2}$$
$$= \sqrt{4+1+1}$$
$$|\bar{a}| = \sqrt{6}$$

$$|\bar{b}| = \sqrt{(1)^2 + (-3)^2 + (-5)^2}$$
$$= \sqrt{1 + 9 + 25}$$
$$|\bar{b}| = \sqrt{35}$$

$$|\bar{c}| = \sqrt{(3)^2 + (-4)^2 + (-4)^2}$$
$$= \sqrt{9 + 16 + 16}$$
$$|\bar{c}| = \sqrt{41}$$

$\therefore$ angle between $\bar{a}$ & $\bar{c}$:

$$\cos\theta = \frac{\bar{a}\cdot\bar{c}}{|\bar{a}|\cdot|\bar{c}|}$$
$$= \frac{(2i - j + k)\cdot(3i - 4j - 4k)}{\sqrt{6}\cdot\sqrt{41}}$$
$$= \frac{(2)(3) + (-1)(-4) + (1)(-4)}{\sqrt{6}\cdot\sqrt{41}}$$

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$$= \frac{6+4-4}{\sqrt{246}}$$

$$\cos\theta = \frac{6}{\sqrt{246}}$$

$$\theta = \cos^{-1}\frac{6}{\sqrt{246}}$$

$$\boxed{\theta = 1.178}$$

angle between $\bar{b}$ & $\bar{c}$,

$$\cos\theta = \frac{\bar{b}\cdot\bar{c}}{|\bar{b}|\cdot|\bar{c}|}$$

$$= \frac{(i-3j-5k)\cdot(3i-4j-4k)}{\sqrt{35}\cdot\sqrt{41}}$$

$$= \frac{(1)(3)+(-3)(-4)+(-5)(-4)}{\sqrt{1435}}$$

$$= \frac{3+12+20}{\sqrt{1435}}$$

$$\cos\theta = \frac{35}{\sqrt{1435}}$$

$$\theta = \cos^{-1}\frac{35}{\sqrt{1435}}$$

$$\boxed{\theta = 1.546}$$

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Q6) Show that $A(4,1,3)$, $B(1,3,2)$, $C(\cancel{3}2,0,7)$ are the vertices of right angled triangle.

Soln: Let $\bar{OA} = 4i + j + 3k$
$\bar{OB} = i + 3j + 2k$
$\bar{OC} = 2i + 7k$

$\therefore \bar{AB} = \bar{OB} - \bar{OA}$
$= (i+3j+2k) - (4i+j+3k)$
$= i + 3j + 2k - 4i - j - 3k$
$\bar{AB} = -3i + 2j - k$

$\bar{BC} = \bar{OC} - \bar{OB}$
$= (2i+7k) - (i+3j+2k)$
$= 2i + 7k - i - 3j - 2k$
$\bar{BC} = i - 3j + 5k$

$\cancel{\bar{AC} = \bar{OC} - \bar{OA}}$
$\cancel{= (2i+7k) - (4i+j+3k)}$
$\cancel{= 2i + 7k - 4i - j - 3k}$
$\cancel{\bar{AC} = -2i - j + 4k}$

$\bar{CA} = \bar{OA} - \bar{OC}$
$= (4i + j + 3k) - (2i + 7k)$
$= 4i + j + 3k - 2i - 7k$
$\bar{CA} = 2i + j - 4k$

$\therefore \bar{AB} \cdot \bar{CA} = (-3i + 2j - k) \cdot (2i + j - 4k)$
$= (-3)(+2) + (2)(+1) + (-1)(-4)$
$= -6 + 2 + 4$

$\boxed{\bar{AB} \cdot \bar{AC} = 0}$

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Q7) Find the angle between $\bar{a} = 2i + 2j - k$ and $\bar{b} = 6i - 3j + 2k$ also find the projection of $\bar{a}$ and $\bar{b}$.

Soln: ~~angle between $\bar{a}$ & $\bar{b}$ =~~
~~$\cos\theta$ =~~

$\bar{a} = 2i + 2j - k$
$\bar{b} = 6i - 3j + 2k$

$$\bar{a}\cdot\bar{b} = (2i + 2j - k)\cdot(6i - 3j + 2k)$$
$$= (2)(6) + (2)(-3) + (-1)(2)$$
$$= 12 - 6 - 2$$
$$\bar{a}\cdot\bar{b} = 4$$

$$|\bar{a}| = \sqrt{(2)^2 + (2)^2 + (-1)^2}$$
$$= \sqrt{4+4+1}$$
$$= \sqrt{9}$$
$$|\bar{a}| = 3$$

$$|\bar{b}| = \sqrt{(6)^2 + (-3)^2 + (2)^2}$$
$$= \sqrt{36 + 9 + 4}$$
$$= \sqrt{49}$$
$$|\bar{b}| = 7$$

$\therefore$ angle between $\bar{a}$ & $\bar{b}$ is,

$$\cos\theta = \frac{\bar{a}\cdot\bar{b}}{|\bar{a}|\cdot|\bar{b}|}$$
$$= \frac{4}{3\cdot 7}$$
$$\cos\theta = \frac{4}{21}$$
$$\theta = \cos^{-1}\frac{4}{21} \qquad \therefore \boxed{\theta = 1.379}$$

~~Q8)~~ projectio

Q8) Find th
$2i - j$

Soln: Let $\bar{a} = 2$
$\bar{b} =$

$\bar{a}\times\bar{b} =$

$|\bar{a}\times\bar{b}| =$
=
=

Let $\hat{e}$

$\hat{e} =$

$= \pm$

$\hat{e} =$

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$3j + 2k$

Q7) projection of $\bar{a}$ on $\bar{b}$ = $\frac{\bar{a}\cdot\bar{b}}{|\bar{b}|}$

$$= \boxed{\frac{4}{7}}$$

Q8) Find the unit vector perpendicular to each vector $2i - j + k$ and $3i + 4j - k$

Soln: Let $\bar{a} = 2i - j + k$
$\bar{b} = 3i + 4j - k$

$$\bar{a}\times\bar{b} = \begin{vmatrix} i & j & k \\ 2 & -1 & 1 \\ 3 & 4 & -1 \end{vmatrix} = i(1-4) - j(-2-3) + k(8+3)$$
$$= -3i + 5j + 11k$$

$$|\bar{a}\times\bar{b}| = \sqrt{(-3)^2 + (5)^2 + (11)^2}$$
$$= \sqrt{9 + 25 + 121}$$
$$= \sqrt{155}$$

Let $\hat{e}$ be the unit vector $\perp$ to $\bar{a}$ & $\bar{b}$

$$\hat{e} = \pm\left(\frac{\bar{a}\times\bar{b}}{|\bar{a}\times\bar{b}|}\right)$$
$$= \pm\left(\frac{-3i + 5j + 11k}{\sqrt{155}}\right)$$

$$\boxed{\hat{e} = \frac{-3i}{\sqrt{155}} + \frac{5j}{\sqrt{155}} + \frac{11k}{\sqrt{155}} \quad \text{or} \quad \frac{3i}{\sqrt{155}} - \frac{5j}{\sqrt{155}} - \frac{11k}{\sqrt{155}}}$$

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Q9) If $\bar{a} = 2i + j + k$, $\bar{b} = i - j - k$, $\bar{c} = 2i - 2j - k$

Find : ① $\bar{a} \cdot (\bar{b} \times \bar{c})$ ④ $(\bar{a} + \bar{b}) \times \bar{c}$

② $(\bar{a} \times \bar{b}) \times \bar{c}$ ⑤ $(\bar{a} - \bar{b}) \times \bar{c}$

③ $(\bar{a} + \bar{b}) \times (\bar{a} - \bar{b})$

Soln: $\bar{a} = 2i + j + k$

$\bar{b} = i - j - k$

$\bar{c} = 2i - 2j - k$

1) $\bar{a} \cdot (\bar{b} \times \bar{c})$

$\therefore \bar{b} \times \bar{c} = \begin{vmatrix} i & j & k \\ 1 & -1 & -1 \\ 2 & -2 & -1 \end{vmatrix} = i(1-2) - j(-1+2) + k(-2+2)$

$= -i - j$

$\therefore \bar{a} \cdot (\bar{b} \times \bar{c}) = (2i + j + k) \cdot (-i - j)$

$= (2)(-1) + (1)(-1) + (1)(0)$

$= -2 - 1 + 0$

$\boxed{\bar{a} \cdot (\bar{b} \times \bar{c}) = -3}$

2) $(\bar{a} \times \bar{b}) \times \bar{c}$

$\bar{a} \times \bar{b} = \begin{vmatrix} i & j & k \\ 2 & 1 & 1 \\ 1 & -1 & -1 \end{vmatrix} = i(-1+1) - j(-2-1) + k(-2-1)$

$= 3j - 3k$

$\therefore (\bar{a} \times \bar{b}) \times \bar{c} = \begin{vmatrix} i & j & k \\ 0 & 3 & -3 \\ 2 & -2 & -1 \end{vmatrix} = i(-3-6) - j(0+6) + k(0-6)$

$= -9i - 6j - 6k$

$\boxed{(\bar{a} \times \bar{b}) \times \bar{c} = -9i - 6j - 6k}$

3) $(\bar{a} + \bar{b}) \times ($

$\bar{a} + \bar{b} = (2$

$= ($

$\bar{a} + \bar{b} =$

$\bar{a} - \bar{b} = ($

$= ($

$\bar{a} - \bar{b} =$

$\therefore (\bar{a} + \bar{b}) \times (\bar{a}$

$(\bar{a} + \bar{b}) \times$

4) $(\bar{a} + \bar{b}) \times$

$(\bar{a} + \bar{b})$

$\therefore (\bar{a} + \bar{b})$

$(\bar{a} + \bar{b})$

5) $(\bar{a} - \bar{b})$

$(\bar{a} - \bar{b}$

$\therefore (\bar{a} - \bar{b}$

$(\bar{a} - \bar{b}$

3) $(\bar{a}+\bar{b})\times(\bar{a}-\bar{b})$

$\bar{a}+\bar{b} = (2i+j+k)+(i-j-k)$
$= (2+1)i+(1-1)j+(1-1)k$
$\bar{a}+\bar{b} = 3i$

$\bar{a}-\bar{b} = (2i+j+k)-(i-j-k)$
$= (2-1)i+(1+1)j+(1+1)k$
$\bar{a}-\bar{b} = i+2j+2k$

$\therefore (\bar{a}+\bar{b})\times(\bar{a}-\bar{b}) = \begin{vmatrix} i & j & k \\ 3 & 0 & 0 \\ 1 & 2 & 2 \end{vmatrix} = i(0+0)-j(6+0)+k(6+0)$
$= -6j+6k$

$\boxed{(\bar{a}+\bar{b})\times(\bar{a}-\bar{b}) = -6j+6k}$

4) $(\bar{a}+\bar{b})\times\bar{c}$

$(\bar{a}+\bar{b}) = 3i$
$\therefore (\bar{a}+\bar{b})\times\bar{c} = \begin{vmatrix} i & j & k \\ 3 & 0 & 0 \\ 2 & -2 & -1 \end{vmatrix} = i(0+0)-j(-3+0)+k(-6+0)$
$= 3j-6k$

$\boxed{(\bar{a}+\bar{b})\times\bar{c} = 3j-6k}$

5) $(\bar{a}-\bar{b})\times\bar{c}$

$(\bar{a}-\bar{b}) = i+2j+2k$
$\therefore (\bar{a}-\bar{b})\times\bar{c} = \begin{vmatrix} i & j & k \\ 1 & 2 & 2 \\ 2 & -2 & -1 \end{vmatrix} = i(-2+4)-j(-1-4)+k(-2-4)$
$= 2i-5j+6k$

$\boxed{(\bar{a}-\bar{b})\times\bar{c} = 2i-5j+6k}$

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Q10) Find the sine angle between vectors $2i - 6j + 3k$ and $4i + 3j - k$

Soln: Let $\bar{a} = 2i - 6j + 3k$

$\bar{b} = 4i + 3j - k$

$\therefore |\bar{a}| = \sqrt{(2)^2 + (-6)^2 + (3)^2}$

$= \sqrt{4 + 36 + 9}$

$= \sqrt{49}$

$|\bar{a}| = 7$

$\therefore |\bar{b}| = \sqrt{(4)^2 + (3)^2 + (-1)^2}$

$= \sqrt{16 + 9 + 1}$

$|\bar{b}| = \sqrt{26}$

$\therefore \bar{a} \times \bar{b} = \begin{vmatrix} i & j & k \\ 2 & -6 & 3 \\ 4 & 3 & -1 \end{vmatrix} = i(6-9) - j(-2-12) + k(6+24)$

$= -3i + 14j + 30k$

$\therefore |\bar{a} \times \bar{b}| = \sqrt{(-3)^2 + (14)^2 + (30)^2}$

$= \sqrt{9 + 196 + 900}$

$|\bar{a} \times \bar{b}| = \sqrt{1105}$

$\therefore \sin\theta = \dfrac{|\bar{a} \times \bar{b}|}{|\bar{a}|\,|\bar{b}|} = \dfrac{\sqrt{1105}}{7 \times \sqrt{26}}$

$\boxed{\sin\theta = 0.931}$

Q11) Determine th...

vectors ...

Soln: Let $\bar{a} = 3i$...

$\bar{b} = 2j$...

Vector area ...

$\therefore \bar{a} \times \bar{b} =$ ...

$\therefore |\bar{a} \times \bar{b}| = \sqrt{\ldots}$

$|\bar{a} \times \bar{b}| =$ ...

$\therefore$ Vector are...

and

Q11) Determine the area of the parallelogram formed by the two vectors ~~2i - 6j + 3~~ $3i + 2j$ and $2j + 4k$

Soln: Let $\bar{a} = 3i + 2j$
$\bar{b} = 2j + 4k$

Vector area of parallelogram $= |\bar{a} \times \bar{b}|$

$$\therefore \bar{a} \times \bar{b} = \begin{vmatrix} i & j & k \\ 3 & 2 & 0 \\ 0 & 2 & 4 \end{vmatrix} = i(8+0) - j(12+0) + k(6+0)$$
$$= 8i - 12j + 6k$$

$$\therefore |\bar{a} \times \bar{b}| = \sqrt{(8)^2 + (-12)^2 + (6)^2}$$
$$= \sqrt{64 + 144 + 36}$$
$$= \sqrt{244}$$
$$|\bar{a} \times \bar{b}| = 2\sqrt{61}$$

$\therefore$ $\boxed{\text{Vector area of parallelogram} = 2\sqrt{61}}$

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Q12) If $\bar{a} = i+j+k$, $\bar{b} = 2i-j-k$, $\bar{c} = 3i+2j-k$
find $(\bar{a}\times\bar{b})\cdot(\bar{a}\times\bar{c})$

Soln: $\bar{a} = i+j+k$
$\bar{b} = 2i-j-k$
$\bar{c} = 3i+2j-k$

$$\therefore \bar{a}\times\bar{b} = \begin{vmatrix} i & j & k \\ 1 & 1 & 1 \\ 2 & -1 & -1 \end{vmatrix} = i(-1+1) - j(-1-2) + k(-1-2)$$
$$= 3j - 3k$$

$$\bar{a}\times\bar{c} = \begin{vmatrix} i & j & k \\ 1 & 1 & 1 \\ 3 & 2 & -1 \end{vmatrix} = i(-1-2) - j(-1-3) + k(2-3)$$
$$= -3i + 4j - k$$

$$\therefore (\bar{a}\times\bar{b})\cdot(\bar{a}\times\bar{c}) = (3j-3k)\cdot(-3i+4j-k)$$
$$= (0)(-3) + (3)(4) + (-3)(-1)$$
$$= 0 + 12 + 3$$
$$\therefore \boxed{(\bar{a}\times\bar{b})\cdot(\bar{a}\times\bar{c}) = 15}$$

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