Tutorial 4

Q1 Find the work done by Force $\bar{F} = 3i - 2j + 4K$ when its point of application moves from $(3, 2, -1)$ to $(2, 5, 4)$

→ Let, Force $= \bar{F} = 3i - 2j + 4K$.

Let $\bar{OA} = 3i + 2j - K$

$\bar{OB} = 2i + 5j + 4K$

$\bar{d} = \bar{AB} = \bar{OB} - \bar{OA}$

$= (2i + 5j + 4K) - (3i + 2j - K)$

$= (2-3)i + (5-2)j + (4+1)K$

$\bar{d} = -i + 3j + 5K$

Work done by Force $\bar{F}$

$W = \bar{F} \cdot \bar{d}$

$= (3i - 2j + 4K) \cdot (-i + 3j + 5K)$

$= (3)(-1) + (-2)(3) + (4)(5)$

$= -3 - 6 + 20$

$W = 11$ units.

Q2 A particle acted upon by forces $\bar{F}_1$ & $\bar{F}_2$ is displaced from $(1,2,3)$ to $(5,4,1)$. Find work done, where $\bar{F}_1 = 4i+j-3k$ and $\bar{F}_2 = 3i+j-k$.

Sol:

Given: $\bar{F}_1 = 4i+j-3k$, $\bar{F}_2 = 3i+j-k$

let $\bar{OA} = i+2j+3k$

$\bar{OB} = 5i+4j+k$.

consider, $\bar{F} = \bar{F}_1 + \bar{F}_2$

$= (4i+j-3k) + (3i+j-k)$

$= (4+3)i + (1+1)j + (-3-1)k$

$\bar{F} = 7i+2j-4k$

$\bar{d} = \bar{AB} = \bar{OB} - \bar{OA}$

$= (5i+4j+k) - (i+2j+3k)$

$= (5-1)i + (4-2)j + (1-3)k$

$\bar{d} = 4i+2j-2k$

Work done by force $\bar{F}$

$W = \bar{F} \cdot \bar{d}$

$= (7i+2j-4k) \cdot (4i+2j-2k)$

$= (7)(4) + (2)(2) + (-4)(-2)$

$= 28+4+8$

$= 40$ units.

3 A particle is displaced from the point whose P.V is $2i - 3j + 4k$ to the point $3i - 2j + k$ under the action of the forces $2i + 4j - 3k$, $5i - 2j + k$, and $i - j + k$. Find the total work done.

Sol. Given:

Let $\bar{F}_1 = 10i - j + 11k$

$\bar{F}_2 = 4i + 5j + 6k$

$\bar{F}_3 = -2i + j - 9k$

Let $\bar{OA} = 5i - 5j - 7k$

$\bar{OB} = 6i + 2j - 2k$

$\bar{F} = \bar{F}_1 + \bar{F}_2 + \bar{F}_3$

$= (10i - j + 11k) + (4i + 5j + 6k) + (-2i + j - 9k)$

$= (10 + 4 - 2)i + (-1 + 5 + 1)j + (11 + 6 - 9)k$

$\bar{F} = 12i + 5j + 8k$.

$\bar{d} = \bar{AB} = \bar{OB} - \bar{OA}$

$= (6i + 2j - 2k) - (5i - 5j - 7k)$

$= (6 - 5)i + (2 + 5)j + (-2 + 7)k$

$\bar{d} = i + 7j + 5k$.

Work done by force $\bar{F}$

$W = \bar{F} \cdot \bar{d}$

$= (12i + 5j + 8k) \cdot (i + 7j + 5k)$

$= (12)(1) + (5)(7) + (8)(5)$

$= 12 + 35 + 40$

$= 87$ units

4 A particle is displaced from the point whose PV is $2\bar{i} - 3\bar{j} + 4\bar{k}$ to the point $3\bar{i} - 2\bar{j} + \bar{k}$ under the action of the forces $2\bar{i} + 4\bar{j} - 3\bar{k}$, $5\bar{i} - 2\bar{j} + \bar{k}$ and $\bar{i} - \bar{j} + \bar{k}$. Find the total work done.

Sol: Given: Let, $\bar{F_1} = 2\bar{i} + 4\bar{j} - 3\bar{k}$

$\bar{F_2} = 5\bar{i} - 2\bar{j} + \bar{k}$

$\bar{F_3} = \bar{i} - \bar{j} + \bar{k}$.

$\bar{F} = \bar{F_1} + \bar{F_2} + \bar{F_3}$

$\bar{F} = (2\bar{i} + 4\bar{j} - 3\bar{k}) + (5\bar{i} - 2\bar{j} + \bar{k}) + (\bar{i} - \bar{j} + \bar{k})$

$= (2+5+1)\bar{i} + (4-2-1)\bar{k} + (-3+1+1)\bar{k}$

$\bar{F} = 6\bar{i} + \bar{j} - \bar{k}$.

let $\bar{OA} = 2\bar{i} - 3\bar{j} + 4\bar{k}$

$\bar{OB} = 3\bar{i} - 2\bar{j} + \bar{k}$

$\bar{d} = \bar{AB} = \bar{OB} - \bar{OA}$

$= (3\bar{i} - 2\bar{j} + \bar{k}) - (2\bar{i} - 3\bar{j} + 4\bar{k})$

$= (3-2)\bar{i} + (-2+3)\bar{j} + (1-4)\bar{k}$

$= \bar{i} + \bar{j} - 3\bar{k}$

Work done by force $\bar{F}$

$W = \bar{F} \cdot \bar{d}$

$= (6\bar{i} + \bar{j} - \bar{k}) \cdot (\bar{i} + \bar{j} - 3\bar{k})$

$= (6)(1) + (1)(1) + (-1)(-3)$

$= 6 + 1 + 3$

$W = 12$

5 Find the work done by the forces represented by the vectors $(1,2,-1)$ and $(3,-5,2)$ acting on a particle which moves from the point $(3,2,1)$ to the point $(4,5,6)$ along the straight line

Sol. Given: consider, $\bar{F_1} = i + 2j - k$

$\bar{F_2} = 3i - 5j + 2k$

$\bar{F} = \bar{F_1} + \bar{F_2}$

$= i + 2j - k + 3i - 5j + 2k$

$\bar{F} = 4i - 3j + k$

consider, $\bar{OA} = 3i + 2j + k$

$\bar{OB} = 4i + 5j + 6k$

$\bar{d} = \bar{AB} = \bar{OB} - \bar{OA}$

~~$= 3i + 2j + k - (4i + 5j + 6k)$~~

~~$= -(3-4) + (2-5)j + ($~~

$= (4i + 5j + 6k) - (3i + 2j + k)$

$= (4-3)i + (5-2)j + (6-1)k$

$\bar{d} = i + 3j + 5k$

work done by force $\bar{F}$

$W = \bar{F} \cdot \bar{d}$

$= (4i - 3j + k) \cdot (i + 3j + 5k)$

$= (4)(1) + (-3)(3) + (1)(5)$

$= 4 - 9 + 5$

$W = 0$ units.

6 Find vector moment of force $2i+2j-4k$ acting at a point $(1,-1,2)$ about the point $(2,-1,3)$

sol:

consider, $\bar{F} = 3i+2j-4k$

$A = (1,-1,2)$

$\bar{OA} = i-j+2k$

$B = (2,-1,3)$

$\bar{OB} = 2i-j+3k$

$\bar{d} = \bar{BA} = \bar{OA} - \bar{OB}$

$= (i-j+2k) - (2i-j+3k)$

$= (1-2)i + (-1+1)j + (2-3)k$

$\bar{d} = -i+0j-k$

movement by the force $\bar{F}$ about the point

$B = \bar{d} \times \bar{F} = (-i+0j-k) \times (3i+2j-4k)$

$$= \begin{vmatrix} i & j & k \\ -1 & 0 & -1 \\ 3 & 2 & -4 \end{vmatrix}$$

$= (0+2)i - (4+3)j + (-2-0)k$

$= 2i-7j-2k$

(7) Three forces $i+2j-3k$, $2i+3j+4k$, $-i-j+k$ are acting on a particle at a point $(2,1,-2)$. Find the moment of the system about the point $(1,1,-1)$.

Sol. Given: consider, $\bar{F_1} = i+2j-3k$
$\bar{F_2} = 2i+3j+4k$
$\bar{F_3} = -i-j+k$.

consider, $A \equiv (2,1,-2)$
$\bar{OA} = 2i+j-2k$
$B \equiv (1,1,-1)$
$\bar{OB} = i+j-k$.

$\bar{F} = \bar{F_1} + \bar{F_2} + \bar{F_3}$
$= (i+2j-3k) + (2i+3j+4k) + (-i-j+k)$
$= (1+2-1)i + (2+3-1)j + (-3+4+1)k$
$\bar{F} = 2i+4j+2k$

$\bar{d} = \bar{BA} = \bar{OA} - \bar{OB}$
$= (2i+j-2k) - (i+j-k)$
$= (2-1)i + (1-1)j + (-2+1)k$
$\bar{d} = i+0j-k$

moment by the force $\bar{F}$ about the point:
$= \bar{d} \times \bar{F}$
$= (i+0j-k) \times (2i+4j+2k)$
$= \begin{vmatrix} i & j & k \\ 1 & 0 & -1 \\ 2 & 4 & 2 \end{vmatrix}$
$= (0+4)i - (2+2)j + (4-0)k$
$= 4i-4j+4k$

(8)

Sol.

(8) Find the magnitude of the moment of a force $-4i+j+k$ acting at a point with P.V $i+2j+3k$ about the point with P.V $-3i+5j+5k$.

Sol. Consider, $\bar{F} = -4i+j+k$

$\bar{OA} = i+2j+3k$

$\bar{OB} = -3i+5j+5k$

$\bar{d} = \bar{BA} = \bar{OA} - \bar{OB}$

$= (i+2j+3k) - (-3i+5j+5k)$

$= (1+3)i + (2-5)j + (3-5)k$

$\bar{d} = 4i-3j-2k$.

movement by the force $\bar{F}$ about the point

$\bar{B} = \bar{d} \times \bar{F}$

$= (4i-3j-2k) \times (-4i+j+k)$

$$= \begin{vmatrix} i & j & k \\ 4 & -3 & -2 \\ -3 & 5 & 5 \end{vmatrix}$$

$= (-15+10)i - (20-6)j + (20-9)k$

$= -5i-14j+11k$.

$|\bar{d} \times \bar{F}| = \sqrt{\quad} -5$

(9) A force of magnitude 3 units in the direction $2i+3j+6k$ act at $(1,1,1)$ Find its movement about the point $(-1,2,3)$

sol. consider, $\bar{a} = 2i+3j+6k$

$|\bar{a}| = \sqrt{2^2+3^2+6^2} = \sqrt{49} = 7$

unit vector $\hat{a} = \frac{2i+3j+6k}{7}$

$\bar{F}$ = force vector of magnitude 3 : $3\times\hat{a}$

$= \frac{3}{7}(2i+3j+6k)$

consider, $A \equiv (1,1,1)$

$B \equiv (-1,2,3)$

$OA = (i+j+k)$

$OB = (-i+2j+3k)$

$\bar{d} = \bar{BA} = (i+j+k) - (-i+2j+3k)$

$= 2i - j - 2k$

moment by force $\bar{F}$ about the point B

$= \bar{d} \times \bar{F}$

$= \frac{3}{7}\begin{vmatrix} i & j & k \\ 2 & -1 & -2 \\ 2 & 3 & 6 \end{vmatrix}$

$= \frac{3}{7}(0i-16j+8k)$

$= \frac{3}{7}\times 8(2j+k)$

$= \frac{24}{7}(2j+k)$

(10) Three forces $2\bar{i}+\bar{j}+\bar{k}$, $-\bar{i}-\bar{j}-\bar{k}$, & $3\bar{i}-2\bar{j}+2\bar{k}$ are acting at a point $(-2,1,2)$ Find the moment of the forces about the point $(1,1,1)$.

Sol. consider, $\bar{F}_1 = 2\bar{i}+\bar{j}+\bar{k}$

$\bar{F}_2 = -\bar{i}-\bar{j}-\bar{k}$

$\bar{F}_3 = 3\bar{i}-2\bar{j}+2\bar{k}$

$\bar{F} = \bar{F}_1+\bar{F}_2+\bar{F}_3$

$= (2\bar{i}+\bar{j}+\bar{k})+(-\bar{i}-\bar{j}-\bar{k})+(3\bar{i}-2\bar{j}+2\bar{k})$

$= (2-1+3)\bar{i}+(1-1-2)\bar{j}+(1-1+2)\bar{k}$

$\bar{F} = 4\bar{i}-2\bar{j}+2\bar{k}$

consider, $A \equiv (-2,1,2)$

$B \equiv (1,1,1)$

$\bar{OA} = (-2\bar{i}+\bar{j}+2\bar{k})$

$\bar{OB} = (\bar{i}+\bar{j}+\bar{k})$

$\bar{d} = \bar{BA} = \bar{OA} - \bar{OB}$

$= (-2\bar{i}+\bar{j}+2\bar{k})-(\bar{i}+\bar{j}+\bar{k})$

$= (-2-1)\bar{i}+(1-1)\bar{j}+(2-1)\bar{k}$

$= -3\bar{i}+0\bar{j}+\bar{k}$

moment by the Force $\bar{F}$ about the point B

$= \bar{d} \times \bar{F}$

$$\begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ -3 & 0 & 1 \\ 4 & -2 & 2 \end{vmatrix}$$

$= (0+2)\bar{i}-(-6-4)\bar{j}+(6-0)\bar{k}$

$= 2\bar{i}+10\bar{j}+6\bar{k}.$