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Tutorial 5

1 $y = 3x^4 + \cos x - \tan x + \sec x$

differentiate y w.r.t x

$$\frac{dy}{dx} = \frac{d}{dx}(3x^4) + \frac{d}{dx}(\cos x) - \frac{d}{dx}(\tan x) + \frac{d}{dx}(\sec x)$$

$$\frac{dy}{dx} = 3(4x^3) + (-\sin x) - \sec^2 x + \sec x \cdot \tan x$$

$$\frac{dy}{dx} = 12x^3 - \sin x - \sec^2 x + \sec x \tan x$$

2 $y = xe^x \cos x$

Given: $y = xe^x \cos x$

differentiate y w.r.t. x

$$\frac{dy}{dx} = \frac{d}{dx}(xe^x \cos x)$$

$$= xe^x \frac{d}{dx}\cos x + x\cos x \frac{d}{dx}(e^x) + e^x \cos x \frac{d}{dx}(x)$$

$$= xe^x(-\sin x) + x\cos x\, e^x + e^x \cos x(1)$$

$$= -\sin x(x)e^x + x\cos x\, e^x + e^x \cos x$$

$$= e^x(-x\sin x + x\cos x + \cos x)$$

3 $y = \frac{2+\sqrt{x}}{2-\sqrt{x}}$

given : $y = \frac{2+\sqrt{x}}{2-\sqrt{x}}$

differentiate w.r.t. x

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{2+\sqrt{x}}{2-\sqrt{x}}\right)$$

$$\frac{dy}{dx} = \frac{(2-\sqrt{x})\frac{d}{dx}(2+\sqrt{x}) - (2+\sqrt{x})\frac{d}{dx}(2-\sqrt{x})}{(2-\sqrt{x})^2}$$

$$= \frac{(2-\sqrt{x})\left(\frac{d}{dx}(2) + \frac{d}{dx}(\sqrt{x})\right) - (2+\sqrt{x})\left(\frac{d}{dx}(2) - \frac{d}{dx}(\sqrt{x})\right)}{(2-\sqrt{x})^2}$$

$$= \frac{(2-\sqrt{x})\left(\frac{1}{2\sqrt{x}}\right) + (2+\sqrt{x})\left(\frac{1}{2\sqrt{x}}\right)}{(2-\sqrt{x})^2}$$

$$= \frac{\frac{2-\sqrt{x}}{2\sqrt{x}} + \frac{2+\sqrt{x}}{2\sqrt{x}}}{(2-\sqrt{x})^2}$$

$$= \frac{\frac{2-\sqrt{x}+2+\sqrt{x}}{2\sqrt{x}}}{(2-\sqrt{x})^2}$$

$$= \frac{\frac{4}{2\sqrt{x}}}{(2-\sqrt{x})^2}$$

$$\frac{dy}{dx} = \frac{2}{\sqrt{x}(2-\sqrt{x})^2}$$

4 $y = \frac{1-x^2}{1+x^2}$

given: $y = \frac{1-x^2}{1+x^2}$

differentiate w.r.t. x

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{1-x^2}{1+x^2}\right)$$

$$\frac{dy}{dx} = \frac{(1+x^2)\frac{d}{dx}(1-x^2) - (1-x^2)\frac{d}{dx}(1+x^2)}{(1+x^2)^2}$$

$$= \frac{(1+x^2)\left(\frac{d}{dx}(1) - \frac{d}{dx}(x^2)\right) - (1-x^2)\left(\frac{d}{dx}(1) + \frac{d}{dx}(x^2)\right)}{(1+x^2)^2}$$

$$= \frac{(1+x^2)(-2x) - (1-x^2)(2x)}{(1+x^2)^2}$$

$$= \frac{-2x-2x^3-(2x-2x^3)}{(1+x^2)^2}$$

$$= \frac{-2x-2x^3-2x+2x^3}{(1+x^2)^2}$$

$$\frac{dy}{dx} = \frac{-4x}{(1+x^2)^2}$$

5 $y = \frac{\sin x}{1-\cos x}$

Given: $y = \frac{\sin x}{1-\cos x}$

differentiate w.r.t. x

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{\sin x}{1-\cos x}\right)$$

$$= \frac{(1-\cos x)\frac{d}{dx}(\sin x) - \sin x\frac{d}{dx}\left(\overset{1-\cos x}{\cancel{\sin x}}\right)}{(1-\cos x)^2}$$

$$= \frac{(1-\cos x)(\cos x) - \sin x(\sin x)}{(1-\cos x)^2}$$

$$= \frac{(1-\cos x)\cos x - \sin^2 x}{(1-\cos x)^2}$$

$$= \frac{\cos x - \cos^2 x - (1-\cos^2 x)}{(1-\cos x)^2}$$

$$= \frac{\cos x - \cos^2 x - 1 + \cos^2 x}{(1-\cos x)^2}$$

$$= \frac{-1+\cos x}{(1-\cos x)^2}$$

$$= \frac{-1(1-\cos x)}{(1-\cos x)^2}$$

$$\frac{dy}{dx} = \frac{-1}{1-\cos x}$$

6 $y = \log_{10} x + \log_x x + \log_{10} 10$

given: $y = \log_{10} x + \log_x x + \log_{10} 10$

differentiate w.r.t. x

$$\frac{dy}{dx} = \frac{d}{dx}\left(\log_{10} x + \log_x x + \log_{10} 10\right)$$

$$= \frac{d}{dx}\left(\frac{\log x}{\log 10}\right) + \frac{d}{dx}(1) + \frac{d}{dx}(1)$$

$$= \frac{d}{dx}\left(\frac{\log x}{\log 10}\right) + 0 + 0$$

$$= \frac{1}{\log 10}\frac{d}{dx}(\log x)$$

$$= \frac{1}{\log 10} \times \frac{1}{x}$$

$$\frac{dy}{dx} = \frac{1}{x \log 10}$$

7 $y = 5a^x + \sqrt[4]{x^3} + \log x^2$

Given: $y = 5a^x + \sqrt[4]{x^3} + \log x^2$

differentiate w.r.t x

$$\frac{dy}{dx} = \frac{d}{dx}(5a^x) + \frac{d}{dx}(\sqrt[4]{x^3}) + \frac{d}{dx}(\log x^2)$$

$$= 5\frac{d}{dx}(a^x) + \frac{d}{dx}(x^{3/4}) + \frac{d}{dx}(2\log x)$$

$$= 5(a^x \log a) + \frac{3x^{-1/4}}{4} + \frac{2}{x}$$

$$\frac{dy}{dx} = 5a^x \log a + \frac{3}{4\sqrt[4]{x}} + \frac{2}{x}$$

8 $y = \left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)^2$

$y = (\sqrt{x})^2 + 2\sqrt{x}\frac{1}{\sqrt{x}} + \left(\frac{1}{\sqrt{x}}\right)^2$

$y = x + 2 + \frac{1}{x}$

differentiate w.r.t. x

$\frac{dy}{dx} = \frac{d}{dx}(x) + \frac{d}{dx}(2) + \frac{d}{dx}\left(\frac{1}{x}\right)$

$\frac{dy}{dx} = 1 + (0) + \left(\frac{-1}{x^2}\right)$

$\frac{dy}{dx} = 1 - \frac{1}{x^2}$

9 $y = \frac{\sin x + \cos x}{\cos x - \sin x}$

Given: $y = \frac{\sin x + \cos x}{\cos x - \sin x}$

differentiate w.r.t. x

$\frac{dy}{dx} = \frac{(\cos x - \sin x)\frac{d}{dx}(\sin x + \cos x) - (\sin x + \cos x)\frac{d}{dx}(\cos x - \sin x)}{(\cos x - \sin x)^2}$

$= \frac{(\cos x - \sin x)\left(\frac{d}{dx}(\sin x) + \frac{d}{dx}(\cos x)\right) - (\sin x + \cos x)\left(\frac{d}{dx}(\cos x) - \frac{d}{dx}(\sin x)\right)}{(\cos x - \sin x)^2}$

$= \frac{(\cos x - \sin x)(\cos x - \sin x) - (\sin x + \cos x)(-\sin x - \cos x)}{(\cos x - \sin x)^2}$

$= \frac{(\cos x - \sin x)^2 \cancel{\cos x} + (\sin x + \cos x)(\sin x + \cos x)}{(\cos x - \sin x)^2}$

$= (\cos x - \sin x)^2 + (\sin x + \cos x)^2 / (\cos x - \sin x)^2$

$= \frac{(\cos x - \sin x)^2}{(\cos x - \sin x)^2} + \frac{(\sin x + \cos x)^2}{(\cos x - \sin x)^2}$

$\frac{dy}{dx} = 1 + \frac{1 + 2\sin x\cos x}{1 - 2\sin x\cos x} = 1 + \frac{1 + \sin 2x}{1 - \sin 2x}$

$$\frac{dy}{dx} = \frac{1-\sin 2x + 1+\sin 2x}{1-\sin 2x}$$

$$\frac{dy}{dx} = \frac{2}{1-\sin 2x}$$

10 $y = \sin^{-1}(\sqrt{x})$

given : $y = \sin^{-1}(\sqrt{x})$

differentiate w.r.t x

$$\frac{dy}{dx} = \frac{d}{dx}\sin^{-1}(\sqrt{x})$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-(\sqrt{x})^2}} \times \frac{d}{dx}(\sqrt{x})$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-x}} \times \frac{1}{2\sqrt{x}}$$

$$\frac{dy}{dx} = \frac{1}{(2\sqrt{x})(\sqrt{1-x})}$$

11 $y = (1+x^2)^{10}$

given : $y = (1+x^2)^{10}$

differentiate w.r.t x

$$\frac{dy}{dx} = \frac{d}{dx}(1+x^2)^{10}$$

$$\frac{dy}{dx} = 10(1+x^2)^9 \frac{d}{dx}(1+x^2)$$

$$\frac{dy}{dx} = 10(1+x^2)^9(2x)$$

$$\frac{dy}{dx} = 20x(1+x^2)^9$$

12 $y = e^{\sin x} + (\log x)^3 + \sqrt{\sin x}$

Given : $e^{\sin x} + (\log x)^3 + \sqrt{\sin x}$, differentiate w.r.t. x

$\frac{dy}{dx} = \frac{d}{dx}(e^{\sin x}) + \frac{d}{dx}(\log x)^3 + \frac{d}{dx}(\sqrt{\sin x})$

$= e^{\sin x}\frac{d}{dx}(\sin x) + 3(\log x)^2\frac{d}{dx}(\log x) + \frac{1}{2\sqrt{\sin x}}\frac{d}{dx}(\sin x)$

$= e^{\sin x}(\cos x) + 3(\log x)^2\left(\frac{1}{x}\right) + \frac{1}{2\sqrt{\sin x}} \times \cos x$

$= e^{\sin x}(\cos x) + \frac{3(\log x)^2}{x} + \frac{\cos x}{2\sqrt{\sin x}}$

$= \cos x\, e^{\sin x} + \frac{3 \times (2 \log x)}{x} + \frac{\cos x}{2\sqrt{\sin x}}$

$\frac{dy}{dx} = \cos x\, e^{\sin x} + \frac{6 \log x}{x} + \frac{\cos x}{2\sqrt{\sin x}}$

13 $y = e^{-7x} \cos 3x$

Given : $y = e^{-7x} \cos 3x$

differentiate w.r.t. x

$\frac{dy}{dx} = (e^{-7x})\frac{d}{dx}(\cos 3x) + \cos 3x \frac{d}{dx}(e^{-7x})$

$= e^{-7x}\left(-\sin 3x \frac{d}{dx}(3x)\right) + \cos 3x\left(e^{-7x}\frac{d}{dx}(-7x)\right)$

$= e^{-7x}(-\sin 3x (3)) + \cos 3x (e^{-7x}(-7))$

$= -3\sin 3x\, e^{-7x} + \cos 3x (e^{-7x}(-7))$

$= -3\sin 3x\, e^{-7x} - 7\cos 3x (e^{-7x})$

$\frac{dy}{dx} = e^{-7x}(-3\sin 3x - 7\cos 3x)$

14 $y = e^{3}\log e^{3\log(x^2+a^2)}$

$= e^{\log(x^2+a^2)^3}$

$y = (x^2+a^2)^3$

differentiate w.r.t. x

$\frac{dy}{dx} = \frac{d}{dx}(x^2) + \frac{d}{dx}($

$\frac{dy}{dx} = \frac{d}{dx}(x^2+a^2)^3$

$= 3(x^2+a^2)^2 \frac{d}{dx}(x^2+a^2)$

$= 3(x^2+a^2)^2(2x+0)$

$= 3(x^2+a^2)^2(2x)$

$\frac{dy}{dx} = 6x(x^2+a^2)^2$

15 $y = \log\left(\sin\left(\frac{x}{2}\right)\right)$

Given $y = \log\left(\sin\left(\frac{x}{2}\right)\right)$

differentiate w.r.t. x

$\frac{dy}{dx} = \frac{d}{dx}\left(\log\left(\sin\left(\frac{x}{2}\right)\right)\right)$

$= \frac{1}{\sin(x/2)} \frac{d}{dx}\left(\sin\left(\frac{x}{2}\right)\right)$

$= \frac{1}{\sin(x/2)}\left(\cos\left(\frac{x}{2}\right)\right)\frac{d}{dx}\left(\frac{x}{2}\right)$

$= \frac{\cos(x/2)}{\sin(x/2)}\left(\frac{1}{2}\right)$

$= \frac{1}{2}\cot(x/2)$

$\frac{dy}{dx} = \frac{\cot(x/2)}{2}$

16 $y = \frac{1}{\cos^2 x \sin^2 x}$

given: $y = \frac{1}{\sin^2 x} \times \frac{1}{\cos^2 x}$

$y = \operatorname{cosec}^2 x \sec^2 x$

differentiate y w.r.t x

$$\frac{dy}{dx} = \operatorname{cosec}^2 x \frac{d}{dx}(\sec^2 x) + \sec^2 x \frac{d}{dx}(\operatorname{cosec}^2 x)$$

$$= \operatorname{cosec}^2 x \left(2\sec x \frac{d}{dx}(\sec x)\right) + \sec^2 x \left(2\operatorname{cosec}^2 x \frac{d}{dx}(\operatorname{cosec} x)\right)$$

$$= \operatorname{cosec}^2 x (2\sec x \cdot \sec x \cdot \tan x) + \sec^2 x (2\operatorname{cosec}^2 x \operatorname{cosec} x \cot x)$$

$$= \operatorname{cosec}^2 x (2\sec^2 x \tan x) + \sec^2 x (2\operatorname{cosec}^2 x \cot x)$$

$$= \frac{1}{\sin^2 x}\left(\frac{2}{\cos^2 x} \times \frac{\sin x}{\cos x}\right) + \frac{1}{\cos^2 x}\left(\frac{2}{\sin^2 x} \times \frac{\cos x}{\sin x}\right)$$

$$= \frac{1}{\sin^2 x \cos^2 x}\left(\frac{2\sin x}{\cos x} + \frac{2\cos x}{\sin x}\right)$$

$$= \frac{2(\sin^2 x + \cos^2 x)}{\sin^3 x \cos^3 x}$$

$$\frac{dy}{dx} = 2\operatorname{cosec}^3 x \sec^3 x$$

17 $y = \tan^{-1}(\cos(2x))$

Given: $y = \tan^{-1}(\cos(2x))$

differentiate w.r.t x

$$\frac{dy}{dx} = \frac{d}{dx}(\tan^{-1}(\cos(2x)))$$

$$= \frac{1}{1+(\cos(2x))^2} \times \frac{d}{dx}(\cos(2x))$$

$$= \frac{1}{1+(\cos(2x))^2}(-\sin(2x))\frac{d}{dx}(2x)$$

$$= \frac{-\sin 2x}{1+(\cos(2x))^2} \times (2)$$

$$= \frac{-2\sin 2x}{1+\cos^2(2x)}$$

$$= \frac{-2(2\sin x\cos x)}{1+\cos^2 x-\sin^2 x}$$

$$= \frac{-4\sin x\cos x}{\cancel{1+\cos^2 x}\ 1-\sin^2 x+\cos^2 x}$$

$$= \frac{-4\sin x\cos x}{\cos^2 x+\cos^2 x}$$

$$= \frac{-4\sin x\cos x}{2\cos^2 x}$$

$$= \frac{-2\sin x}{\cos x}$$

$$\frac{dy}{dx} = -2\tan x$$

18 $y = \cos(xe^x)$

Given $y = \cos(xe^x)$

differentiate w.r.t x

$$\frac{dy}{dx} = \frac{d}{dx}(\cos(xe^x))$$

$$= -\sin(xe^x)\frac{d}{dx}(xe^x)$$

$$= -\sin(xe^x)\left(x\frac{d}{dx}(e^x) + e^x\frac{d}{dx}(x)\right)$$

$$= -\sin(xe^x)\left(xe^x + e^x(1)\right)$$

$$= -\sin(xe^x)(e^x(x+1))$$

$$\frac{dy}{dx} = -e^x(x+1)\sin(xe^x)$$

19 $y = x\sin^{-1}(x) + \sqrt{1-x^2}$

Given: $y = x\sin^{-1}x + \sqrt{1-x^2})$

differentiate w.r.t x

$$\frac{dy}{dx} = \frac{d}{dx}(x\sin^{-1}x + \sqrt{1-x^2})$$

$$= \frac{d}{dx}(x\sin^{-1}x) + \frac{d}{dx}(\sqrt{1-x^2})$$

$$= \left(x\frac{d}{dx}\sin^{-1}x + \sin^{-1}x\frac{d}{dx}(x)\right) + \frac{1}{2\sqrt{1-x^2}}\frac{d}{dx}(1-x^2)$$

$$= x\frac{1}{\sqrt{1-x^2}} + \sin^{-1}x(1) + \frac{1}{2\sqrt{1-x^2}}(-2x)$$

$$= \frac{x}{\sqrt{1-x^2}} + \sin^{-1}x - \frac{\cancel{2}x}{\cancel{2}\sqrt{1-x^2}}$$

$$\frac{dy}{dx} = \sin^{-1}x$$

20 $y = \sin^2(e^{3x})$

given: $y = \sin^2(e^{3x})$

differentiate w.r.t x

$$\frac{dy}{dx} = \frac{d}{dx}\left(\sin^2(e^{3x})\right)$$

$$\frac{dy}{dx} = 2\sin(e^{3x})\frac{d}{dx}\left(\cancel{e^{3x}}\ \sin(e^{3x})\right)$$

$$= 2\sin(e^{3x})\cos(e^{3x})\frac{d}{dx}(e^{3x})$$

$$= 2\sin(e^{3x})\cos(e^{3x})(e^{3x})\frac{d}{dx}(3x)$$

$$= \sin(2e^{3x})(e^{3x})(3)$$

$$\frac{dy}{dx} = 3e^{3x}\sin(2e^{3x})$$