22/2/25

Page No.
Date

Tutorial 7

1 $\int \frac{1}{\sqrt[4]{(2-3x)^2}} dx$

Let $I = \int \frac{dx}{(2-3x)^{2/4}}$

$= \int (2-3x)^{-1/2} dx$

$= \left(\frac{(2-3x)^{-1/2+1}}{-1/2+1}\right)\frac{1}{3} + c$

$= \frac{2}{3}(2-3x)^{1/2} + c$

2 $\int \left(\frac{1}{1+x^2} - \frac{\cos x}{\sin^2 x}\right) dx$

$I = \int \left(\frac{1}{1+x^2} - \frac{\cos x}{\sin^2 x}\right) dx$

$= \int \frac{1}{1+x^2} dx - \int \frac{\cos x}{\sin^2 x} dx$

$= \int \frac{dx}{x^2+1} - \int \operatorname{cosec} x \cdot \cot x \, dx$

$= \tan^{-1} x - (-\operatorname{cosec} x) + c$

$\Rightarrow \tan^{-1} x + \operatorname{cosec} x + c$

$= \operatorname{cosec} x + \tan^{-1} x + c$

3 $\int x(x-1)^2 dx$

Let $I = \int x(x-1)^2 dx$

$= \int x(x^2 - 2x + 1) dx$

$= \int (x^3 - 2x^2 + x) dx$

$= \int x^3 dx - 2\int x^2 + \int x\, dx$

$= \frac{x^{3+1}}{3+1} - 2\left(\frac{x^{2+1}}{2+1}\right) + \frac{x^{1+1}}{1+1} + C$

$= \frac{x^4}{4} - 2\frac{x^3}{3} + \frac{x^2}{2} + C$

$= \frac{x^4}{4} - \frac{2}{3}x^3 + \frac{x^2}{2} + C$

4 $\int \frac{1+x-x^2}{\sqrt{x}} dx$

Let $I = \int \frac{1+x-x^2}{\sqrt{x}} dx$

$= \int \left(\frac{1}{\sqrt{x}} + \frac{x}{\sqrt{x}} - \frac{x^2}{\sqrt{x}}\right) dx$

$= \int \frac{dx}{\sqrt{x}} + \int \sqrt{x}\, dx - \int x^{3/2} dx$

$= 2\sqrt{x} + \frac{x^{3/2}}{3/2} - \frac{x^{5/2}}{5/2}$

$= 2\sqrt{x} + \frac{2x^{3/2}}{3} - \frac{2x^{5/2}}{5} + C$

5 $f'(x) = 3x^2 - 4x + k$, $f(0) = 1$, $f(2) = 11$, $f(x)$

Let $f(x) = \int f'(x)\,dx$

$= \int (3x^2 - 4x + k)\,dx$

$= 3\int \cancel{3}x^2\,dx - 4\int x + \int k\,dx$

$= 3\left(\frac{x^{2+1}}{2+1}\right) - 4\left(\frac{x^{1+1}}{1+1}\right) + kx + c$

$= 3\left(\frac{x^3}{3}\right) - 4\left(\frac{x^2}{2}\right) + kx + c$

$f(x) = x^3 - 2x^2 + kx + c$

$f(0) = 1$ — (Given)

put $x = 0$.

$\therefore f(0) = 0^3 - 2(0)^2 + k(0) + c$

$\boxed{1 = c}$

put $x = 2$

$f(2) = (2)^3 - 2(2)^2 + k(2) + c$

$11 = 8 - 8 + 2k + 1$

$11 - 1 = 2k$

$k = \frac{10}{2}$

$\boxed{k = 5}$

substituting.

$f(x) = x^3 - 2x^2 + 5x + 1$

6 $f'(x) = bx^3 + 3x^2 + x - 1$, $f(0) = 2$, $f(3) = 5$, $f(x)$

$f(x) = \int f(x)\,dx$

$= \int (bx^3 + 3x^2 + x - 1)\,dx$

$= \int bx^3\,dx + \int 3x^2\,dx + \int x\,dx - \int dx$

$= b\int \frac{x^{3+1}}{3+1} + 3\left(\frac{x^{2+1}}{2+1}\right) + \frac{x^{1+1}}{1+1} - x + c$

$= \frac{bx^4}{4} + x^3 + \frac{x^2}{2} - x + c$

put $x = 0$.

$f(0) = \frac{b(0)}{4} + 0^3 + \frac{0^2}{2} - 0 + c$

$\boxed{2 = c}$

put $x = 3$

$f(3) = \frac{b(3)^4}{4} + 3^3 + \frac{3^2}{2} - 3 + c$

$5 = b\frac{81}{4} + 27 + \frac{9}{2} - 3 + c$

$5 = b\frac{81}{4} + \frac{24 \times 2 + 9}{2} + 2$

$3 - \frac{48 + 9}{2} = \frac{81b}{4}$

$3 - \frac{57}{2} = \frac{81}{4}b$

$\frac{6 - 57}{2} = \frac{81}{4}b$

$-\frac{51}{2} \times \frac{4}{81} = b$

$b = -\frac{34}{27}$

$f(x) = -\frac{34}{27}x^4 + 4x^3 + 2x^2 - 4x +$

Page No.
Date

7 $\int \sin^2(2x)\,dx$

Let $I = \int \sin^2(2x)\,dx$

$= \frac{1}{2}\int 2\sin^2(2x)\,dx$

$= \frac{1}{2}\int (1 - \cos(4x))\,dx$

$= \frac{1}{2}\left(\int dx - \int \cos(4x)\,dx\right)$

$= \frac{1}{2}\left(x + \frac{\sin 4x}{4}\right) + c$

$= \frac{x}{2} + \frac{\sin 4x}{8} + c$

8 $\int \cos^2 x$

Let $I = \int \cos^2 x$

$= \frac{1}{2}\int 2\cos^2 x$

$= \frac{1}{2}\int (1 + \cos(4x))\,dx$

$= \frac{1}{2}\left(\int dx + \int \cos(4x)\,dx\right)$

$= \frac{1}{2}\left(x + \frac{\sin 4x}{4}\right) + c$

$= \frac{x}{2} + \frac{\sin 4x}{8} + c$

Page No.
Date

9 $\int (\tan x + \cot x)^2 dx$

Let $I = \int (\tan^2 x + 2\tan x \cot x + \cot^2 x) dx$

$= \int (\tan^2 x + 2 + \text{cosec}^2 x - 1) dx$

$= \int (\sec^2 x - 1 + 2 + \text{cosec}^2 x$

$= \int (\sec^2 x + \text{cosec}^2 x) dx$

$= \int \sec^2 x dx + \int \text{cosec}^2 x dx$

$= \tan x - \cot x + C$

10 (1) $\int \sin^3 x dx$

Let $I = \int \sin^3 x dx$

$= \int \frac{3\sin x - \sin 3x}{4} dx$

$= \frac{1}{4}\left[3\int \sin x - \int \sin 3x dx\right]$ 6

$= \frac{1}{4}\left[-3\cos x + \frac{\cos 3x}{3}\right] + C$

$= -\frac{3}{4}\cos x + \frac{\cos 3x}{3} + C$

(2) $\int \cos^3 x dx$

Let $I = \int \cos^3 x dx$

$= \int \frac{\cos 3x - 3\cos x}{4} dx$

$= \frac{1}{4}\left[\int \cos 3x dx - 3\int \cos x dx\right]$

$= \frac{1}{4}\left[\frac{\sin 3x}{3} - 3\sin x\right]$

$= \frac{\sin 3x}{12} - \frac{3\sin 3x}{4}$

11 $\int \frac{\cos 2x}{\sin^2 x \cos^2 x} dx$

Let $I = \int \frac{\cos 2x}{\sin^2 x \cos^2 x} dx$

$= \int \frac{\cos^2 x - \sin^2 x}{\sin^2 x \cos^2 x} dx$

$= \int \left(\frac{\cos^2 x}{\sin^2 x \cos^2 x} - \frac{\sin^2 x}{\sin^2 x \cos^2 x} \right) dx$

$= \int (\operatorname{cosec}^2 x - \sec^2 x) dx$

$= \int \operatorname{cosec}^2 x \, dx - \int \sec^2 x \, dx$

$-\cot x - \tan x + c.$

12 $\int \frac{dx}{(x+1)(x+2)(x+3)}$

Let $I = \int \frac{dx}{(x+1)(x+2)(x+3)}$

consider: $\frac{1}{(x+1)(x+2)(x+3)} = \frac{A}{(x+1)} + \frac{B}{(x+2)} + \frac{C}{(x+3)}$

$1 = A(x+2)(x+3) + B(x+1)(x+3) + C(x+1)(x+2) \rightarrow (1)$

put $x = -1$ in eq. (1)

$1 = A(-1+2)(-1+3)$

$1 = A(1)(2)$

$2A = 1$

$\boxed{A = 1/2}$

put $x = -2$ in eq. (1)

$1 = B(-2+1)(-2+3)$

$1 = -B$

$\boxed{B = -1}$

put $x = -3$

$$1 = C(-3+1)(-3+2)$$

$$1 = 2C$$

$$\boxed{C = 1/2}$$

Substituting,

$$I = \int \left(\frac{1}{2(x+1)} - \frac{1}{(x+2)} + \frac{1}{2(x+3)} \right) dx$$

$$= \int \frac{1}{2x+2}\, dx - \int \frac{1}{x+2}\, dx + \int \frac{1}{2x+6}\, dx$$

$$= \frac{1}{2} \log|2x+2| - \log|x+2| + \frac{1}{2} \log|2x+6| + C$$

13

13 $\int \frac{3x^2+2}{x^3-4x}\,dx$

Let $I = \int \frac{3x^2+2}{x^3-4x}\,dx$

consider: $\frac{3x^2+2}{x^3-4x} = \frac{3x^2+2}{x(x^2-4)} = \frac{3x^2+2}{x(x+2)(x-2)}$

$= \frac{3x^2+2}{x(x+2)(x-2)} = \frac{A}{x} + \frac{B}{x+2} + \frac{C}{x-2}$

$3x^2+2 = A(x+2)(x-2) + B(x)(x-2) + C(x)(x+2) \Rightarrow (1)$

put $x=0$ in eq. (1).

$3(0)^2+2 = A(0+2)(0-2)$

$2 = -4A$

$\boxed{A = -1/2}$

put $x=2$ in eq. (1).

$3(2)^2+2 = C(2)(2+2)$

$14 = 8C$

$\boxed{C = 7/4}$

put $x = -2$

$$3(-2)^2 + 2 = B(-2)(-2-2)$$

$$14 = 8B$$

$$\boxed{B = 7/4}$$

substituting:

$$\int \left(\frac{-1}{2x} + \frac{7}{4(x+2)} + \frac{7}{4(x-2)} \right) dx$$

$$= \frac{-1}{2} \int \frac{dx}{\cancel{2}x} + 7 \int \frac{dx}{4x+8} + 7 \int \frac{dx}{4x-8}$$

c

$$= \frac{-1}{2} \log|x| + \frac{7}{4} \log|4x+8| + \frac{7}{4} \log|4x-8| + c$$

$$= \frac{1}{2} \left(\frac{7}{2} \log|4x+8| + \frac{7}{2} \log|4x-8| - \log|x| \right) + c$$

14 $\int \frac{x^2}{(x+2)^2 (x-1)} dx$

Let $I = \int \frac{x^2}{(x+2)^2(x-1)} dx$

consider : $\frac{x^2}{(x+2)(x+2)(x-1)} = \frac{A}{(x+2)} + \frac{B}{(x+2)^2} + \frac{C}{(x-1)}$

$x^2 = A(x+2)(x-1) + B(x-1) + C(x+2)(x+2) \rightarrow 1$

put $x=1$ in eq. 1 .

$1^2 = C(1+2)(1+2)$

$1 = 9C$

$\boxed{C = 1/9}$

put $x = -2$ in eq. 2 .

$(-2)^2 = B(-2-1)$

$4 = -3B$

$\boxed{B = -4/3}$

put $x=0$

$0 = A(2)(-1) + B(-1) + C(2)(2)$

$0 = -2A + \frac{4}{3} + \frac{4}{9}$

$0 = -2A + \frac{12+4}{9}$

$2A = \frac{16}{9}$

$\boxed{A = \frac{8}{9}}$

substituting

$$= \int \frac{8}{9(x+2)} - \frac{4}{3(x+2)^2} + \frac{1}{9(x-1)} \, dx$$

$$= 8\int \frac{dx}{9x+18} - \frac{4}{3}\int \frac{dx}{(x+2)^2} + \int \frac{dx}{9x-9} \, dx$$

$$= \frac{8}{9} \log|9x+18| - \frac{4}{3}\left(\frac{-1}{x+2}\right) + \frac{1}{9} \log|9x-9| + c$$

$$= \frac{1}{9}\left(8\log|9x+18| + \frac{12}{x+2} + \log|9x-9|\right) + c$$

15 $\int \frac{x^2+1}{x(x-1)^2}$

Let $I = \int \frac{x^2+1}{x(x-1)^2}$

consider: $\frac{x^2+1}{x(x-1)^2} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{(x-1)^2} \rightarrow$

$x^2+1 = A(x-1)(x-1) + B(x-1)x + C(x) \rightarrow$ 1

put $x = 0$

$0^2+1 = A(-1)(-1)$

$\boxed{A = 1}$

put $x = +1$

$(+1)^2+1 = C(1)$

$\boxed{2 = C}$

put $x = -1$

$(-1)^2 + 1 = 4A + 2B - C$

$2 = 4(1) + 2(B) - 2$

$1 = 1 + B$

$\boxed{B = 0}$

$$\int\left(\frac{1}{x}+\frac{2}{(x-1)^2}\right)dx$$

$$=\int\frac{dx}{x}+2\int\frac{dx}{(x-1)^2}$$

$$=\log|x|+2\left(\frac{-1}{x-1}\right)+c$$

$$=\log|x|-\frac{2}{x-1}+c$$

16 $\int\frac{x^2+1}{(x+1)(x+2)(x-3)}$

Let $I=\int\frac{x^2+1}{(x+1)(x+2)(x-3)}$

consider: $\frac{x^2+1}{(x+1)(x+2)(x-3)}=\frac{A}{x+1}+\frac{B}{x+2}+\frac{C}{x-3}$

$x^2+1=A(x+2)(x-3)+B(x+1)(x-3)+C(x+1)(x+2) \rightarrow (1)$

put $x=-1$

$(-1)^2+1=A(-1+2)(-1-3)$

$2=-4A$

$\boxed{A=-1/2}$

put $x=-2$

$(-2)^2+1=B(-2+1)(-2-3)$

$5=-5B$

$\boxed{B=-1}$

put $x=3$

$3^2+1=C(3+1)(3+2)$

$10=20C$

$\boxed{C=1/2}$